Nonlinear mechanical and poromechanical analyses: comparison with analytical solutions

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Outline

- Introduction
- Theoretical background and COMSOL implementation
- Case 1 : verification of a nonlinear mechanical behavior
- Case 2 : poromechanical behavior – one-dimensional consolidation
- Case 3 : poroelastic behavior of a borehole
- Concluding remarks

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Theoretical background and COMSOL implementation

Fluid-mechanical interaction

$$\begin{cases} -\nabla \underline{\underline{\sigma}} = \rho_{und}\,\boldsymbol{g} = \left(\rho_f \phi + \rho_d\right)\boldsymbol{g} \\ \underline{\underline{\sigma}}' - \underline{\underline{\sigma}}_0' = \underline{\underline{\underline{\underline{C}}}}_0\left(\underline{\underline{\varepsilon}} - \underline{\underline{\varepsilon}}^p\right) \\ \underline{\underline{\sigma}}' - \underline{\underline{\sigma}}_0' = \underline{\underline{\sigma}} - \underline{\underline{\sigma}}_0 + b\left(p - p_0\right)\underline{\underline{I}} \\ \rho_f S \dfrac{\partial p}{\partial t} + \nabla.\rho_f\underbrace{\left[-\dfrac{k_{int}}{\mu_f}\left(\nabla p + \rho_f \boldsymbol{g}\right)\right]}_{\text{Darcy's law}} = \underbrace{-\rho_f b \dfrac{\partial\left(tr\underline{\underline{\varepsilon}}\right)}{\partial t}}_{M \rightarrow H} + Q_m \end{cases}$$

Equilibrium equation

Mechanical behavior

Biot effective stresses (**H** → **M**)

Fluid diffusivity equation

Nonlinear mechanical behavior

The framework of plasticity theory characterized by:

- ❑ a yield function, $\mathbf{F_s}$
- ❑ a hardening/softening function and flow rule, **G** (plastic potential) governing direction and magnitude of strain increment)

$$d\underline{\underline{\sigma}}' = \left[\underline{\underline{\underline{\underline{C}}}}_0 - \frac{\left(\underline{\underline{\underline{\underline{C}}}}_0 : \dfrac{\partial F_s}{\partial \underline{\underline{\sigma}}}\right) \otimes \left(\underline{\underline{C}} : \dfrac{\partial F_s}{\partial \underline{\underline{\sigma}}}\right)}{\dfrac{\partial F_s}{\partial \underline{\underline{\sigma}}} : \underline{\underline{C}} : \dfrac{\partial F_s}{\partial \underline{\underline{\sigma}}} - \dfrac{\partial G}{\partial \gamma}\dfrac{\partial G}{\partial q}}\right] : d\underline{\underline{\varepsilon}}$$

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Verification of nonlinear mechanical behavior (1/3)

Problem statement

To determine the field of stresses and displacements around a cylindrical hole (gallery) in an infinite elastoplastic medium subjected to an initial in situ stresses (isotropic & anisotropic)

- ❑ Failure surface follows the Drucker-Prager criterion (inner adjusting of Mohr-Coulomb pyramid) → **C, ϕ**
- ❑ Plastic potential : associated flow rule (maximum of dilatancy) → **no additional parameters**

Closed-form solution (Salençon 1968)

The solution assumes a Mohr-Coulomb elastoplastic medium and an isotropic initial stress. The plastic radius, r_p, is given by:

$$r_p = r_0 \left(\frac{2}{N_\varphi + 1} \frac{\sigma_0 + \frac{2C\sqrt{N_\varphi}}{N_\varphi - 1}}{p_i + \frac{2C\sqrt{N_\varphi}}{N_\varphi - 1}} \right)^{\frac{1}{N_\varphi - 1}}$$

- ❑ r_0 : hole radius. σ_0 : magnitude of isotropic in situ stresses
- ❑ p_i : internal pressure (assumed to be zero in this example)

$$N_\varphi = \frac{1 + \sin\varphi}{1 - \sin\varphi}$$

Analytical expressions of tangential and radial stresses, radial displacement are proposed in the two domains:

- ❑ elastic domain $r > r_p$
- ❑ plastic domain $r < r_p$

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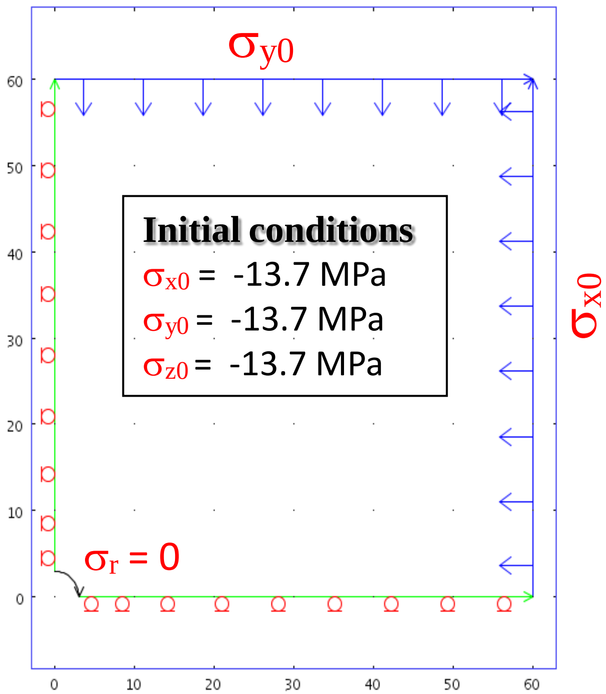
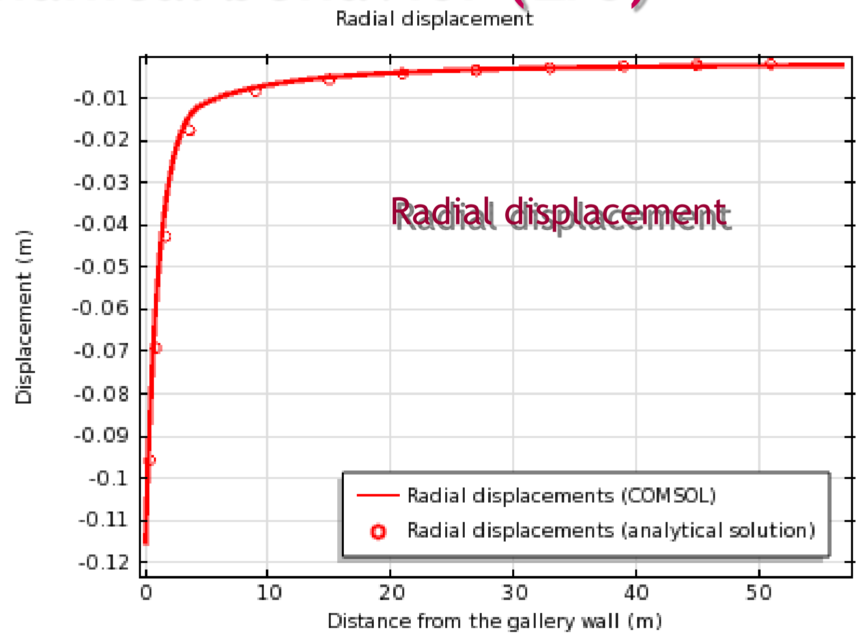
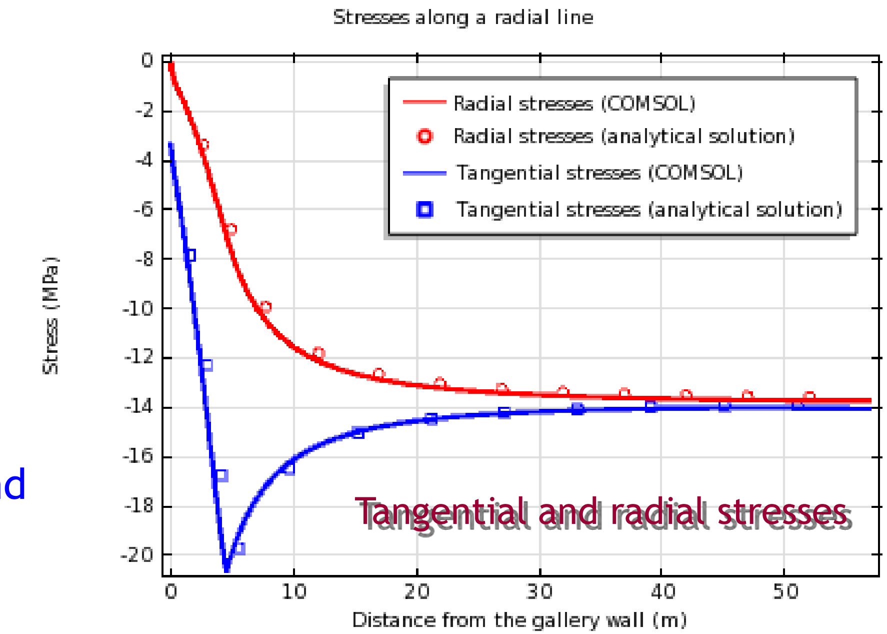
Verification of nonlinear mechanical behavior (2/3)

Analysis of results : isotropic stresses

Initial and boundary conditions







❑ Note a very good agreement between numerical and analytical results

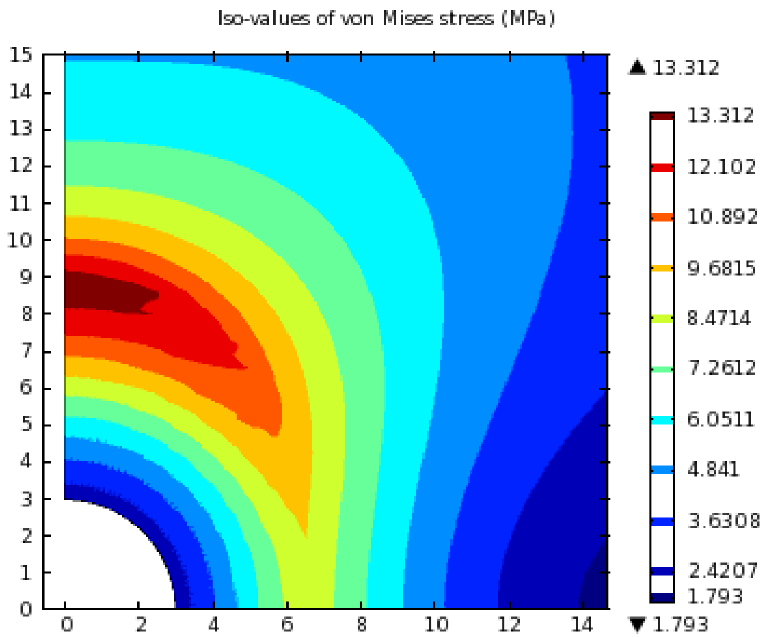
Verification of nonlinear mechanical behavior (3/3)

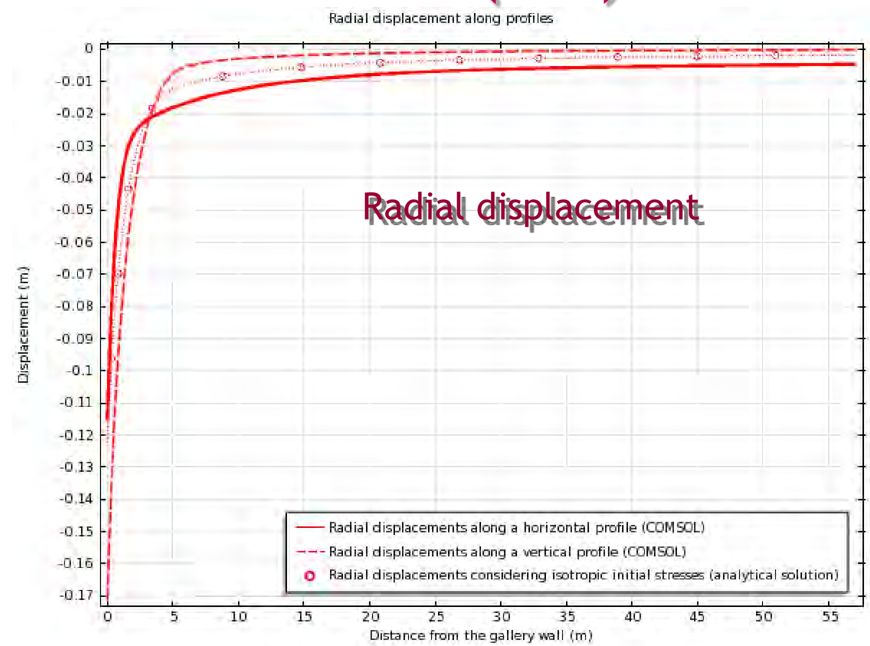
Analysis of results : anisotropic stresses

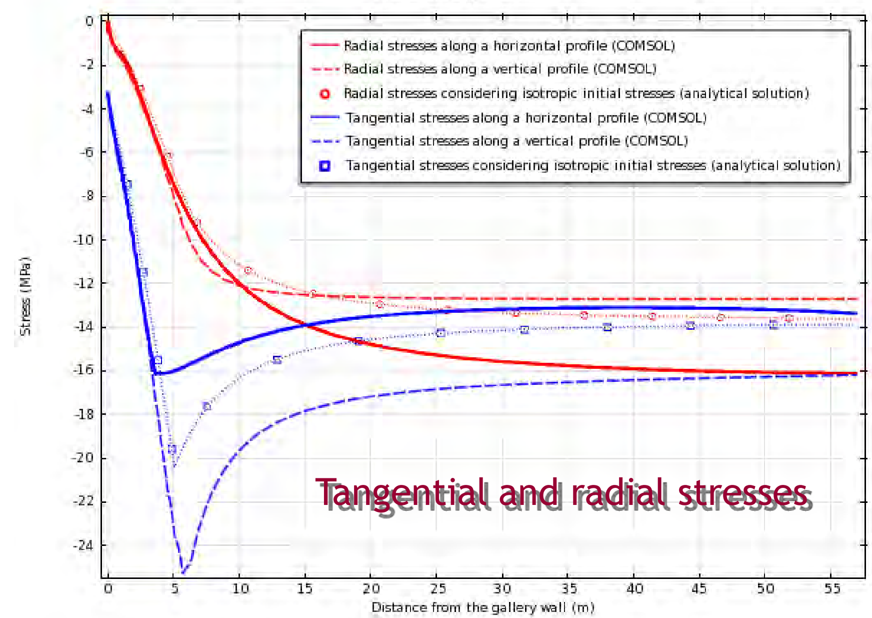
Initial conditions

Initial conditions
σ_{x0} = -16.1 MPa
σ_{y0} = -12.4 MPa
σ_{z0} = -12.7 MPa

Contours of von Mises deviatoric stress







- ❑ The evolution of the predicted curves is qualitatively in adequacy with that we already analytically discuss

Poromechanical verification : 1D consolidation (1/2)

Problem statement

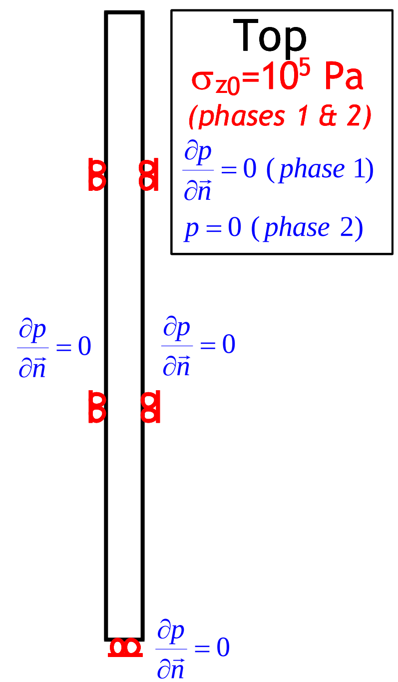
Classical one-dimensional consolidation of a saturated poroelastic column of soil:

- ❑ the soil matrix (squeleton) is homogeneous and behaves elastically,
- ❑ Darcy's transport law is assumed.

Closed-form solution (Detournay & Cheng 1993)

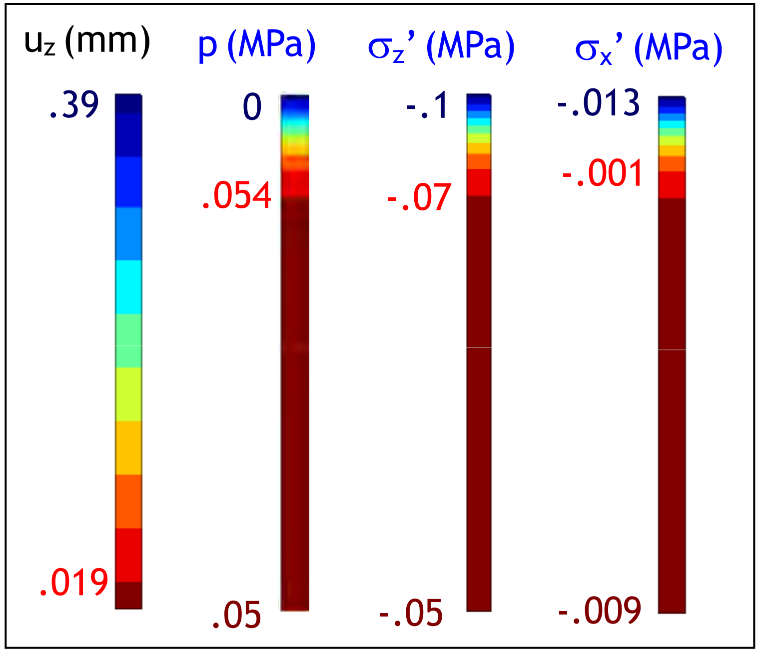
The analytical solution in terms of subsidence, pore pressure and effective stresses to this problem is derived by solving the previous poromechanical system in 1D

Initial and boundary conditions



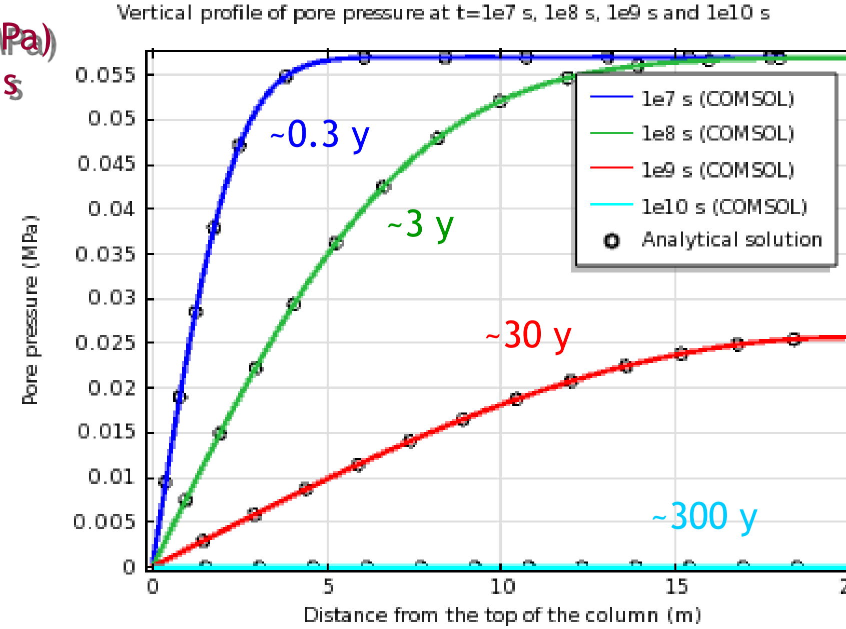
Poromechanical verification : 1D consolidation (2/2)

Analysis of results

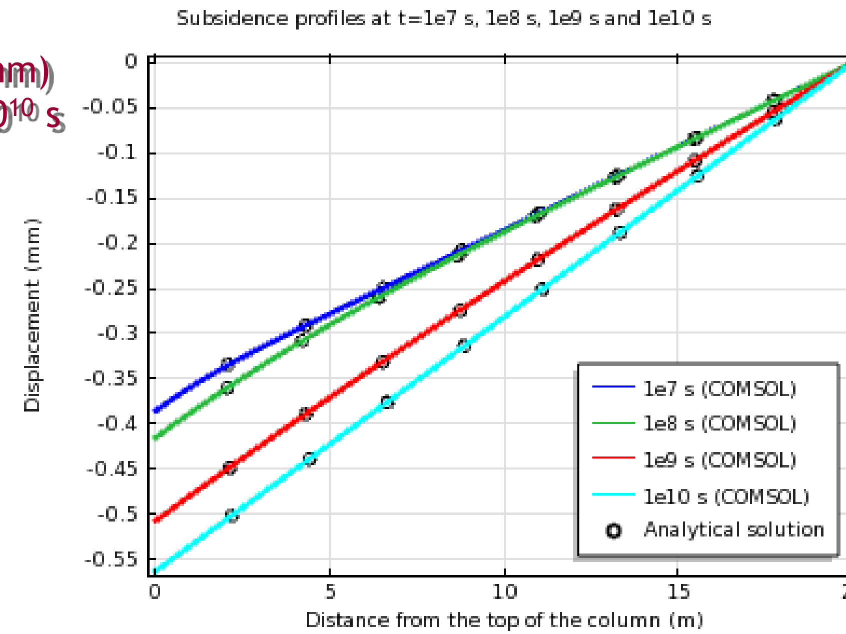


Contours (phases 1 & 2) at t=10^7 s (0.3 y)

Pore pressure (MPa) for t=10^7 to 10^{10} s



Subsidence (mm) for t=10^7 to 10^{10} s



The simulation results are in very good agreement with the analytical solution for:

- pore water pressure dissipation
- time-dependent subsidence

Poromechanical verification : 2D consolidation (1/3)

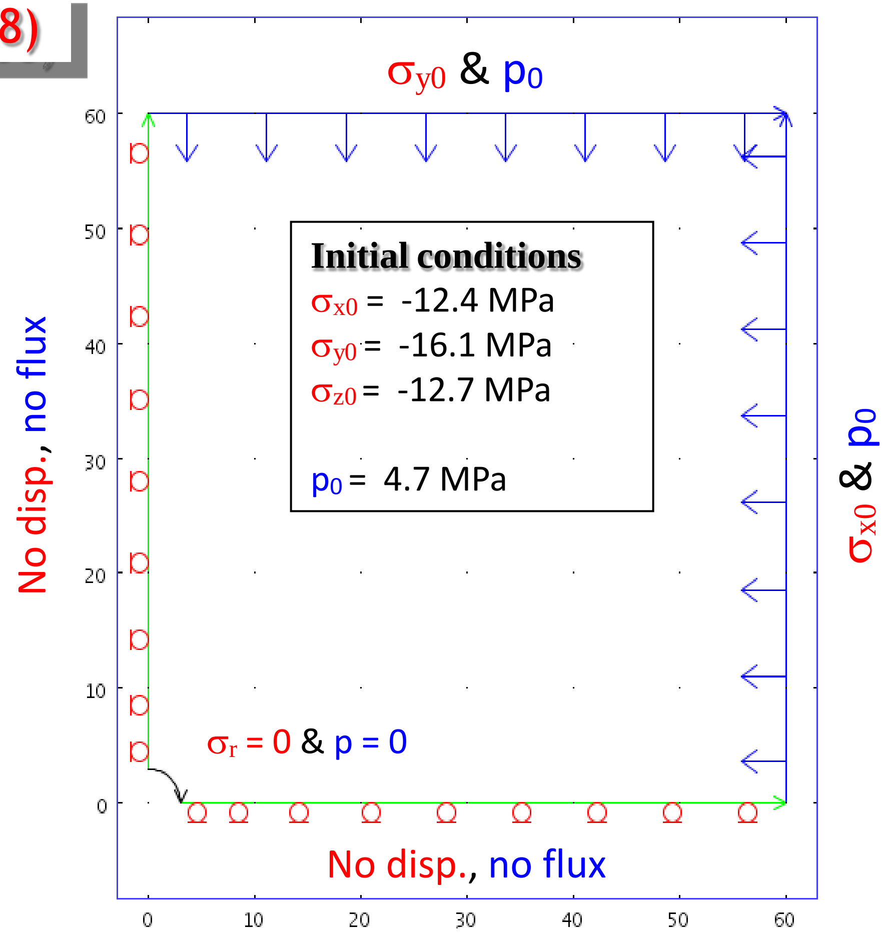
Problem statement

A cylindrical borehole is excavated in a saturated porous rock subject to an anisotropic in situ stress field.

Closed-form solution (Detournay & Cheng 1988)

The analytical solution of this problem is proposed by Detournay and Cheng. The solution is formulated by superposition of asymptotic solutions for three loading modes:

- (1) a far-field isotropic stress (Lamé solution);
- (2) an initial pore pressure distribution;
- (3) a far-field stress deviator



Initial and boundary conditions

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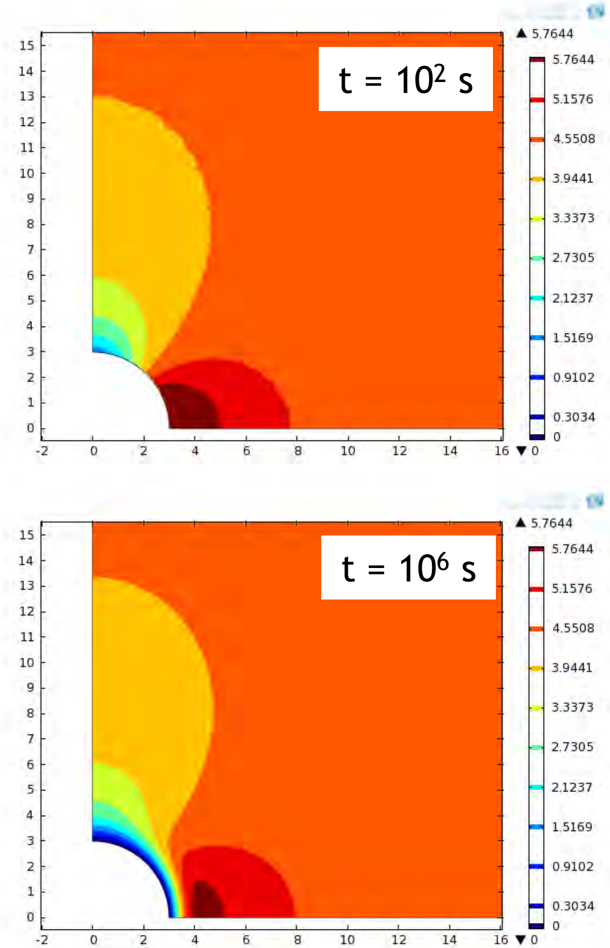
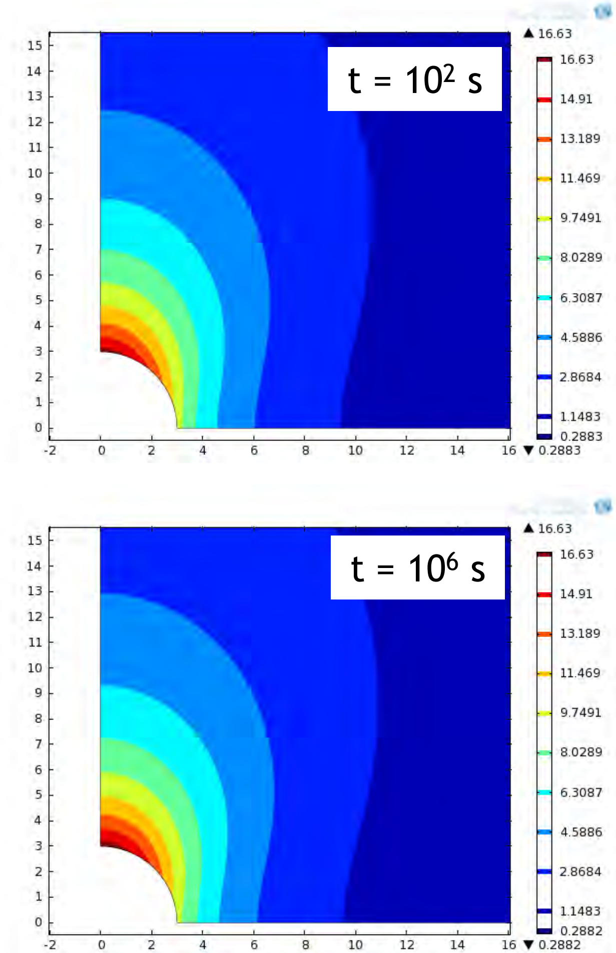
Poromechanical verification : 2D consolidation (2/3)

Numerical results and comparison with analytical solution



Pore pressure (MPa)

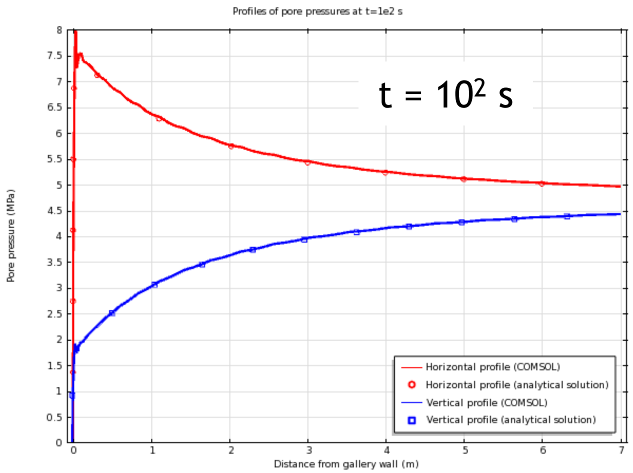
Due to the instantaneous undrained response in an anisotropic in situ stresses, **over-pressures develop in the direction of the initial minor stress**, and **under-pressures in the direction of the initial major stress**, in accordance with the analytical solution.

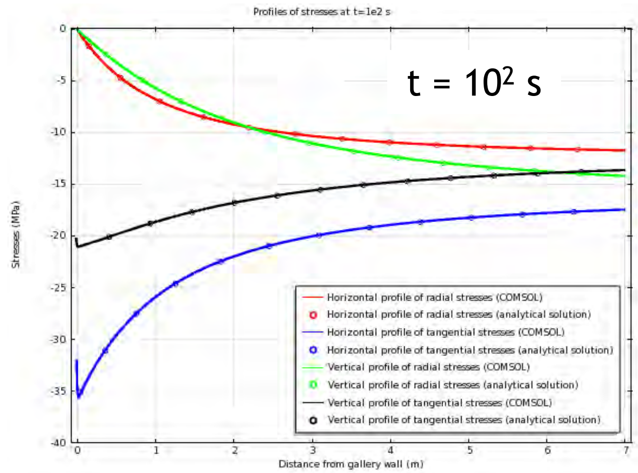


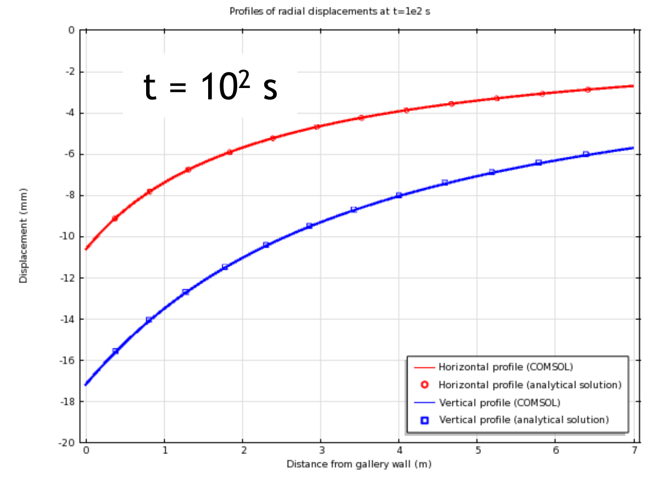
Total displcamement (mm)

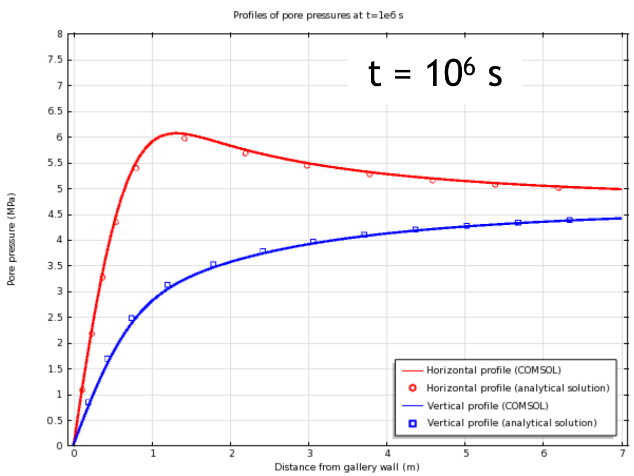
Poromechanical verification : 2D consolidation (3/3)

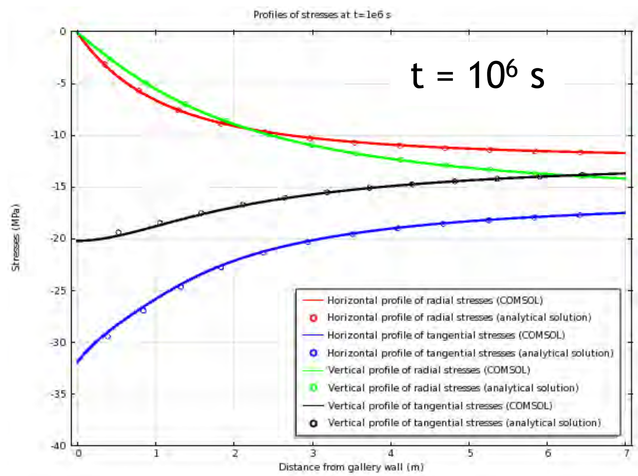
Numerical results and comparison with analytical solution

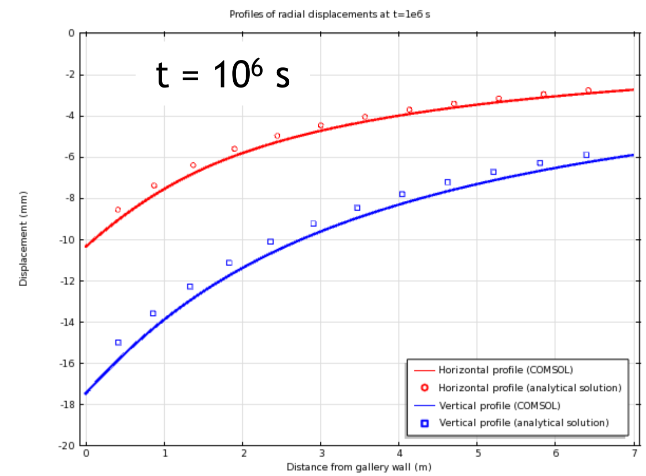












Pore pressure (MPa) | Total stresses (MPa) | Radial displacement (mm)

- ❑ Note a very good agreement between numerical and analytical results

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Concluding remarks

This paper presents an exercise of validation where numerical simulations were performed with COMSOL through three cases of verification

(1) a nonlinear mechanical behavior in the framework of plasticity

(2) a fluid-mechanical interaction in 1D

(3) a fluid-mechanical interaction in 2D

Compared to the closed-form solutions, numerical results are in very good agreement with the analytical ones.

Next stage of this work concerns the following applications:

(a) study of water effects on the stability of slopes and underground cavities,

(b) dimensioning of CO_2 storage sites.

These applications require complementary developments (such as poroplasticity of saturated and unsaturared porous media) which will have to also be validated.

See also poster session

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