Design Optimization of an Electronic Component with an Evolutionary Algorithm Using the COMSOL-MATLAB LiveLink

Eva Pelster[1],David Wenger[1]
[1] Wenger Engineering GmbH, Einsteinstr. 55, 89077 Ulm,
mail@wenger-engineering.com

Abstract: Electric construction components exposed to alternating high voltage have to withstand a significant amount of thermal loads and, due to the changes in temperature, structural stresses. In order to minimize these loads, optimizing the geometry can be a helpful tool in the design process.
In this study, COMSOL Multiphysics is used to predict thermal and mechanical loads on a high-voltage component. An Evolutionary Algorithm is implemented in MATLAB and an optimization process initialized via the COMSOL-MATLAB-Live Link – using the simulated COMSOL results as the optimization variable.

Keywords: Optimization, Evolutionary Algorithm, Joule Heating, Thermal Deformation

1. Introduction

With today's computation resources, it is possible to use an FEM-simulation as a part of a larger investigation, i.e. a parametric study, to gather more knowledge about a simulation model.
Where there are a couple of variable parameters, parametric studies can quickly become time-consuming. In these cases an optimization search is expected to find a good solution faster.
When trying to improve the design of a highly developed component, using an optimization algorithm can provide a new solution, or give an indication of a possible better solution.
Alongside the classical mathematical optimizers – often based on gradients – there are the so-called evolutionary algorithms, which implement mechanisms by mimicking biological evolution.
In the following study, such an evolutionary algorithm has been applied to optimize the geometric design of an electric component, to achieve a desired maximum temperature while limiting the amount of stress resulting from changes in temperature.

Figure 1 shows the initial geometry. The part has a length of 50 mm, a width of 10 mm and a height of 0.3 mm. The geometry represents a thermal fuse and is enclosed in a casing which is not considered in the following calculations.

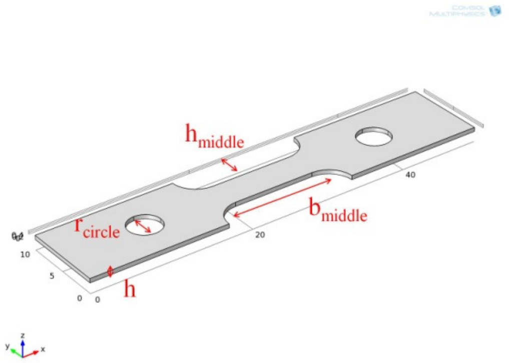

Figure 1 Original Geometry

The smaller side faces are connected to the electronic circuit and are considered to have a constant thermal resistance with regard to the ambient temperature.
Now it is desired to change the geometry in such a way that a defined maximum temperature is achieved at a specified current pulse, while minimizing the resulting structural stresses.
Some geometry parameters are variable in the search for a better suitable geometry; an overview is included in Table 1.

Table 1 Boundaries for variable parameters

Parameter	Lower bound	Upper bound
r_{Circle}	1 mm	4.5 mm
h_{middle}	1 mm	4 mm
b_{middle}	5 mm	15 mm
h	0.1 mm	0.7 mm

2. The Optimization Model

To be able to incorporate the evaluated COMSOL Multiphysics-simulation within the optimization process, COMSOL is executed with the LiveLink for MATLAB. This way the MATLAB-based evolutionary algorithm can easily be run within one user interface, evaluating each solution with the according FEM-simulation by accessing COMSOL.

2.1 The Evolutionary Algorithm

Evolutionary algorithms (EA) represent a class of algorithms which generate solutions based on a search applying techniques such as selection mutation and crossover – imitating biological evolution. To judge the quality of one solution, the so-called fitness value is introduced: The algorithm tries to minimize this value.

Here a genetic algorithm (GA) is used. The concept of GA was developed by Holland et al in the 1960s and 1970s. In the terminology of EA, a solution vector $x \in X$ is called an individual (or *chromosome*) which again is made of discrete units called *genes*, with each gene controlling features of the chromosome. Typically one chromosome corresponds to one unique solution x.

The GA runs with a multitude of chromosomes forming a *population*. The initial population is usually generated randomly. As the search goes on, the population should include fitter solutions.

GA makes use of two evolutionary operators: *crossover* and *mutation*. In crossover generally two chromosomes form a new chromosome: the offspring. The parents are selected based on preference towards a fitter chromosome. With the mutation operator, random changes are applied to genes, bringing diversity back into the population. The procedure of the GA is given by the following steps:

Step 1: Set $n = 1$. Generate N solutions, forming the first population P. Evaluate fitness of solutions.

Step 2: Crossover: Generate offspring Population Q_n.

Step 3: Mutation: Mutate each solution $x \in Q_t$ with a defined mutation rate.

Step 4: Evaluate fitness of each solution$x \in Q_t$.

Step 5: Selection: Select N solutions from Q_t considering their fitness and copy them to P_{n+1}.

Step 6: If stopping criterion is satisfied, terminate search, else, set $n = n + 1$ and continue at Step 2.

2.2 The COMSOL Model

The COMSOL model to be evaluated by the Evolutionary Algorithm should give the temperature distribution and the structural stresses, both resulting from an electrical current flow, as output values.

Therefore the applied physics include Joule Heating and Solid Mechanics.

The component consists of copper, the material properties are listed in Table 2.

Table 2 Material properties for copper

Spec. el. conductivity σ [m/(Ohm mm²)]	58
Heat conductivity λ [W/(m K)]	390
Density ρ [g/cm³]	8.89
Heat capacity at const. pressure c_p [J/(kg K)]	385
Coefficient of thermal expansion α [10^{-6}/K]	17
Young’s Modulus E [kN/mm²]	115
Poisson’s ratio ν [-]	0.35

2.2.1 Joule Heating Model

In the joule heating process the temperature increases due to resistive heating from an electric current. The generated heat $\boldsymbol{Q}$ is proportional to the square of the current density magnitude.

$$\boldsymbol{Q} \propto |\boldsymbol{J}|^2 \qquad (2.1)$$

The electric resistivity ρ = 1/σ (σ denotes electric conductivity) is the coefficient of proportionality. The current density is proportional to the electric field, which in turn equals the negative gradient of the electric potential V.

$$\boldsymbol{J} = \boldsymbol{\sigma E} \qquad (2.2)$$

$$\boldsymbol{E} = \boldsymbol{\nabla V} \qquad (2.3)$$

The heat source $\boldsymbol{Q}$ incorporated into the heat equation yields the mathematical model for heat transfer in solids:

$$\boldsymbol{\rho C_p \frac{\partial T}{\partial t}} = \boldsymbol{\nabla(\lambda \cdot \nabla T) + Q} \qquad (2.4)$$

These equations are applied in the Joule Heating physics in COMSOL. Except for the two smaller lateral faces, all boundaries are considered to be thermally and electronically isolated:

$$-\boldsymbol{n} \cdot \boldsymbol{J} = \boldsymbol{0}, \qquad (2.5)$$

$$-\boldsymbol{n}(-\boldsymbol{\lambda}\nabla \boldsymbol{T}) = \boldsymbol{\alpha} \cdot (\boldsymbol{T_0} - \boldsymbol{T}) \qquad (2.6)$$

In the heat conduction mode at the two lateral faces, a heat transfer resistance between the faces and the ambient temperature $\boldsymbol{T_0}$ is applied. One of the lateral faces is ground ($\boldsymbol{V} = \boldsymbol{0}$) in the electric currents mode while the other one is exposed to a normal inward current density ($-\boldsymbol{n} \cdot \boldsymbol{J} = \boldsymbol{J_{in}}$), the pulse $\boldsymbol{I}(\boldsymbol{t})$ is shown in Figure 2. $\boldsymbol{J}$ is computed by dividing $\boldsymbol{I}$ with the area of the surface to which the boundary condition is applied.

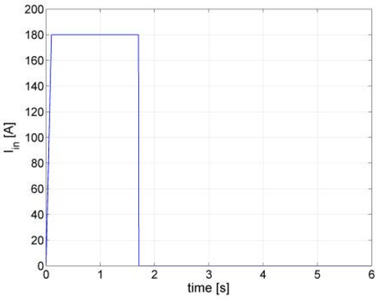

Figure 2 Inward current $\boldsymbol{I_{in}}$

2.2.2 Solid Mechanics Model

On top of the resistive heating, the resulting thermal stresses are calculated within the Solid Mechanics mode.
The relation between the stress $\boldsymbol{\sigma}$ and the strain $\boldsymbol{\varepsilon}$ is given by

$$\boldsymbol{\sigma} = \boldsymbol{D}\boldsymbol{\varepsilon_{el}} = \boldsymbol{D}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon_{th}}) \qquad (2.7)$$

where $\boldsymbol{\varepsilon_{el}}$ and $\boldsymbol{\varepsilon_{th}}$ are the elastic and thermal stresses. $\boldsymbol{D}$ is the $\boldsymbol{6} \times \boldsymbol{6}$ elasticity matrix.

$$D = \frac{E}{(1+\nu)(1-2\nu)} \cdot \begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ \nu & 1-\nu & \nu & 0 & 0 & 0 \\ \nu & \nu & 1-\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-2\nu}{2} \end{bmatrix} \qquad (2.8)$$

And the strain is computed with

$$(\varepsilon \text{ - } \varepsilon_{\text{th}}) = \begin{bmatrix} \varepsilon_x - \alpha(T - T_{ref}) \\ \varepsilon_y - \alpha(T - T_{ref}) \\ \varepsilon_z - \alpha(T - T_{ref}) \\ \varepsilon_{xy} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \end{bmatrix} \qquad (2.9)$$

where $\boldsymbol{T_{ref}}$is the stress-free reference Temperature.

All boundaries are allowed to displace freely except for two edges, which are considered to be fixed, so that no movement in any direction is allowed. This restriction is applied to the one edge of each of the two smaller lateral faces.

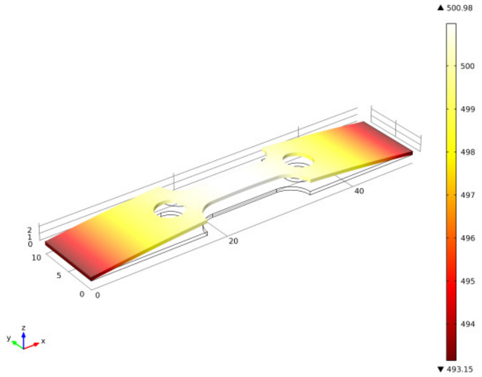

Figure 3 Temperature and deformation at $\boldsymbol{t} = \boldsymbol{1.5}\ \boldsymbol{s}$ for original geometry

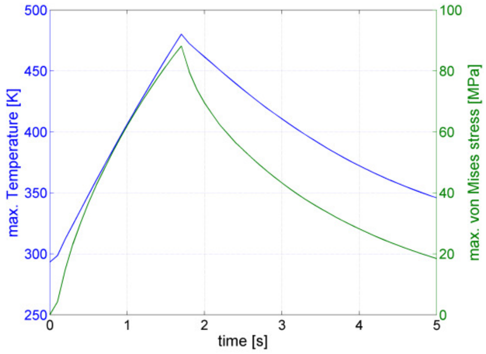

Figure 4 Max. temperature and stress for original geometry

Figure 3 shows the temperature distribution at $t = 1.5\,s$, during a current pulse, the geometry is deformed disproportionately, multiplying the real deformation due to thermal stresses with a factor. Figure 4 shows a plot displaying the temperature in the middle of the geometry (the hottest point) and the overall maximum of the von Mises stress field. These values are accounted for, when calculating the fitness value of every new geometry.

3. The Optimization Procedure

In order to combine the GA with the FEM solution, an appropriate representation of a solution and its corresponding fitness value has to be selected.

In this case each solution is represented by its geometry, varying with different length for the geometric parameters. Each geometric parameter would be one gene of the chromosome.

The fitness value is a mathematical combination of the absolute difference between the maximum temperature in the middle of the geometry and the maximum overall von Mises stress. The objective is to minimize this value.

Figure 5 shows the general cycle in a flow chart. COMSOL is executed at each blue highlighted process step, evaluating each individual's geometry and giving the temperature and von Mises stress field as output values, making it possible to calculate the fitness value.

The GA is run with genetic operators and conditions as defined in Table 2.

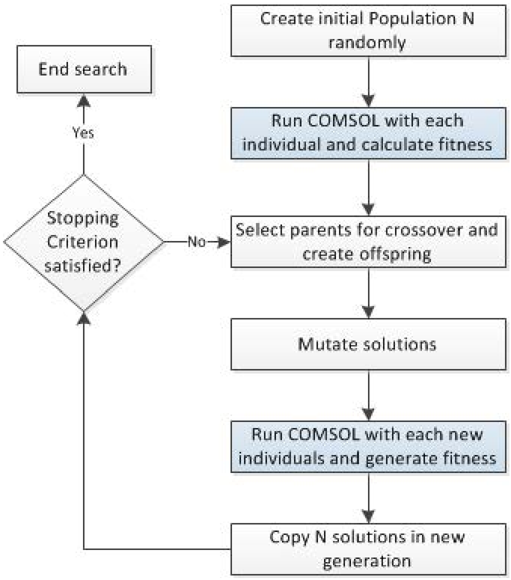
Create initial Population N randomly
Run COMSOL with each individual and calculate fitness
Select parents for crossover and create offspring
Mutate solutions
Run COMSOL with each new individuals and generate fitness
Copy N solutions in new generation
Stopping Criterion satisfied?
No
Yes
End search

Figure 5 Flow chart of optimization process

Table 3 Conditions for GA

Operator	Value
Population size N	20
Crossover probability	0.25
Mutation probability	0.2
Max. generations	400

4. Results

The GA is run with the previously-desribed settings until a defined number of generations is reached. Figure 6 shows the fitness value for the best individual of each generation. It can be seen that there is an overall trend towards a better fitness value, while still maintaining a diverse solution field. In Figure 7 the different individuals (marked red in Figure 6) are displayed. It can clearly be seen how the geometry evolves during the search. Here geometry 4 also represents the final chosen geometry. Here the final maximum temperature is closest to the desired maximum temperature in the component. The behavior for this configuration during a current pulse can be seen in Figure 7.

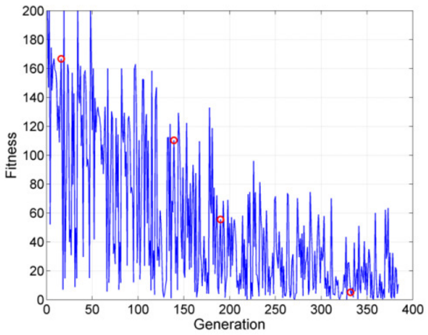

Figure 6 Best fitness value for each generation

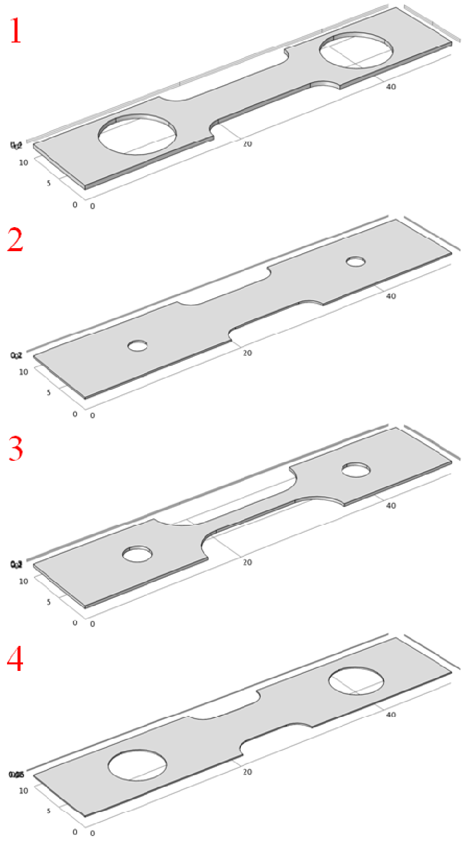

Figure 7 Different configurations during the run of the GA

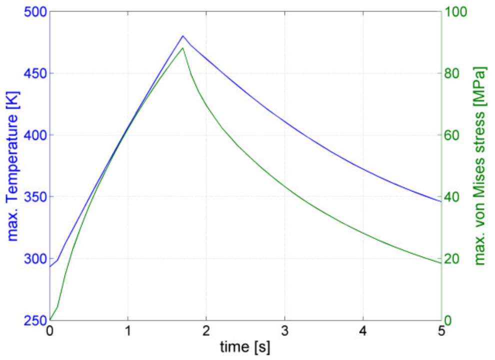

Figure 8 Max. temperature and stress for final geometry

5. Conclusion

It has been shown how the MATLAB LiveLink for COMSOL has been used to implement a Genetic Algorithm. The search was successful in finding a better solution regarding the desired maximum temperature in an electric component while achieving a lower structural stress.

One could imagine using this procedure for a multitude of problems i.e. when a parametric study would be too time-consuming.

8. References

1. Holland, John H, *Adaptation in Natural and Artificial Systems*, University of Michigan Press, Ann Arbor (1975)

2. Goldberg David E., *Genetic algorithms in search, optimization, and machine learning*, Addison-Wesley, Reading (1989)

3. Schwefel, Hans-Paul, *Numerische Optimierung von Computer-Modellen*, Birkhäuser (1977)

4. Michalewicz, Zbigniew, Genetic Algorithms + Data Structures = Evolution Programs, Srpinger (1999)