Implementation of an Isotropic Elastic-Viscoplastic Model for Soft Soils using COMSOL Multiphysics®

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Abstract: In this paper a elastic-viscoplastic (creep) model is implemented in COMSOL v.4.2a and v.4.3 and benchmarked against another commercial finite element software package with a very similar material model. It is also validated against commonly performed laboratory tests such as Constant Rate of Strain oedometer tests (CRS) and K_0-Consolidated Undrained triaxial tests (K_0CU). The implementation in COMSOL is conducted by using a fully coupled analysis between the Solid Mechanics node, Darcy´s Law node and a distributed ODE node. The ODE node is used to simulate the creep contribution. The model implemented provides a material model that works as anticipated and can capture many important features of soft soil behaviour.

Keywords: Soft soil, creep, elastic-viscoplastic model, implementation, benchmark.

1. Introduction

Time-dependent behaviour in clay constitutes an engineering challenge in road design and construction in areas with deep deposits of soft clay. Soil improvement and construction of building foundations or embankments can be quite complicated and expensive in such areas. Construction costs need to be balanced against high maintenance costs. In order to do this optimally, there is a need to predict long-term settlement with a higher degree of accuracy. To be able to predict long-term settlements of building foundations or embankments constructed on soft soils it is necessary to include the effect of creep.

In this paper a elastic-viscoplastic (EVP) model is implemented in COMSOL. The implemented material model EVP is benchmarked against another commercial finite element software package, Plaxis BV, with a very similar material model called the Soft Soil Creep model (SSC). It is also validated against commonly performed laboratory tests such as Constant Rate of Strain oedometer tests (CRS) and K_0-Consolidated Undrained triaxial tests (K_0CU).

2. The isotropic elastic-viscoplastic model

The implemented material model in COMSOL is based on a material model presented by [1].

Some basic characteristics of the EVP model are:

- Stress-dependent stiffness (logarithmic compression behaviour)
- Distinction between primary loading and unloading-reloading
- Creep behaviour
- Memory of preconsolidation pressure
- Failure behaviour according to Matsuoka-Nakai (EMN) criterion

The EVP model is, like the SSC model, based on the modified cam-clay type ellipses. The well-known stress invariants for the mean effective stress, p', and deviatoric stress, q, are adopted, [2]. These stress invariants are used to define the size of the ellipse and stress-strain relation, see eq. (1)-(4) below.

The EVP model is an elastic-viscoplastic model, formulated as a relationship between stress rates and total strain rates. Total strain rates are decomposed into elastic strain rates and creep strain rates as formulated in eq. (1).

$$\dot{\underline{\varepsilon}} = \dot{\underline{\varepsilon}}^e + \dot{\underline{\varepsilon}}^c = \underline{\underline{D}}^{-1}\dot{\underline{\sigma}}' - \frac{1}{\alpha}\frac{1}{r_s \cdot \tau}\left(\frac{p^{eq}}{p_p^{eq}}\right)^{r_s \cdot (\lambda^* - \kappa^*)}\frac{\partial p^{eq}}{\partial \underline{\sigma}'} \quad (1)$$

In eq. (1) $\underline{\underline{D}}^{-1}$ is Hooke´s law of isotropic elasticity with a linear stress-dependent stiffness where the Youngs modulus for unloading/reloading, E_{ur}, is defined as:

$$E_{ur} = 3(1 - 2\nu_{ur})\frac{p'}{\kappa^*} \quad (2)$$

and:

$$\alpha = \frac{\partial p^{eq}}{\partial p'} \qquad (3)$$

$$p^{eq} = p' + \frac{q^2}{M_{cs}^2 p'} \qquad (4)$$

p_{eq} is the equivalent isotropic effective stress based on the Modified Cam-Clay yield function.

The material model comprises of the following parameters:

κ^* = Modified swelling index

λ^* = Modified compression index

r_s = creep number

ν_{ur} = Poission´s ratio for unloading/reloading

τ = Reference time (normally set to 1 day)

p_p^{eq} = Isotropic preconsolidation stress

In Figure 1 the soil parameter M_{CS} and M_{MC} are shown and represent the so-called 'critical state line' and the Mohr-Coulomb failure line.

In this implementation the Mohr-Coulomb failure criteria has been substituted with the extended Matsuoka-Nakai (EMN) failure criterion, see [8]. This has been done to get a smooth failure surface and also that the EMN also uses the intermediate stress unlike the Mohr-Coulomb failure criteria. The implemeted failure criteria is Lode angle dependent, see [3]. The parameters needed to describe the failure criteria is the friction angle (φ), dilatancy angle (ψ) and effective cohesion (c´). The failure surface and reference surface (cap) are visualized in principal stress space in Figure 2.

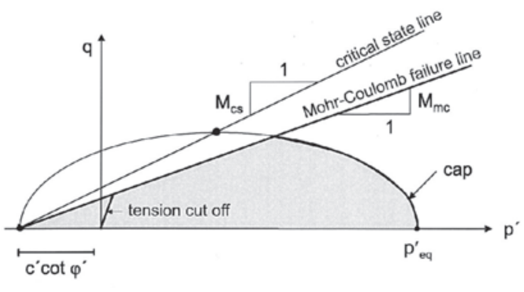

Figure 1. Diagram of p^{eq} ellipse in a p-q plane, [4].

Figure 1 suggests that tensile stresses are possible but this could be prevented by using a tension cut-off option.

The critical state line M_{cs} is calculated according to eq. (5), see [2]. This gives the shape of the cap as shown in Figure 1.

$$M = 3\sqrt{\frac{\left(1-K_0^{nc}\right)^2}{\left(1+2K_0^{nc}\right)^2} + \frac{\left(1-K_0^{nc}\right)\left(1-2\nu_{ur}\right)\left(\frac{\lambda^*}{\kappa^*}-1\right)}{\left(1+2K_0^{nc}\right)\left(1-2\nu_{ur}\right)\frac{\lambda^*}{\kappa^*}-\left(1-K_0^{nc}\right)\left(1+\nu_{ur}\right)}} \qquad (5)$$

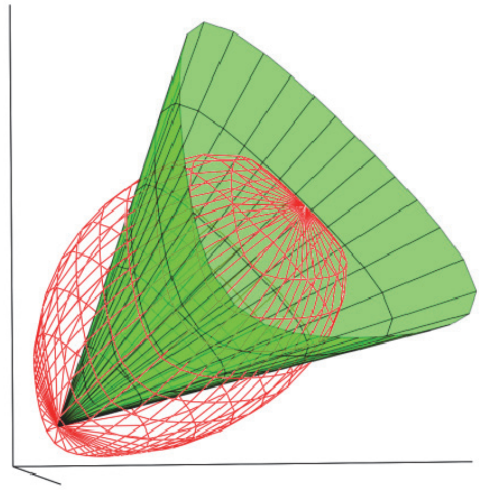

Figure 2. The failure surface (green) and reference surface (red) visualized in principal stress space.

The evolution of the preconsolidation stress, p_p^{eq}, is controlled by the volumetric creep strains:

$$p_p^{eq} = p_{p0}^{eq} \cdot \exp\left(\frac{\Delta\varepsilon_v^c}{\lambda^* - \kappa^*}\right) \qquad (6)$$

Where

$$\dot{\varepsilon}_v^c = \frac{1}{r_s \cdot \tau}\left(\frac{p^{eq}}{p_p^{eq}}\right)^{r_s\left(\lambda^*-\kappa^*\right)} \qquad (7)$$

This expression defines the time-dependent creep behaviour and implies that the isotropic overconsolidation ratio (OCR = p_p^{eq} / p^{eq}) has a considerable influence on the creep rate. If the stress remains unchanged the creep process continues and the preconsolidation stress keeps on increasing, but at a decresing rate. For more details on the SSC model see i.e. [1, 2, 5, 7]

3. Implementation in COMSOL

Since COMSOL has the linear elastic model built in, with the option of isotropic, orthotropic and full anisotropy, the only equation that needs to be implemented in COMSOL is the second part of eq. (1) i.e. the creep part. This has been implemented in Comsol with the use of a distributed ODE node.

The equations, eq.(3)-(6), in addition to derivitives are defined under local varibles.

A Darcy law node is added to couple the pore pressure response that is derived from the volumtric strains or used to set boundary conditions.

The input parameters are given as constants or as user defined functions that could depend on stress or strain for example. This option gives the user total freedom, e.g. over how the preconsolidation stress changes with depth.

4. Benchmark

To validate the implementation of the EVP model a benchmark with the SSC model has been done. The benchmark consists of two K_0CU[1] compression tests, positive in deviatoric stress, and two K_0CU extension tests, negative in deviatoric stress.

Input parameters that have been used are presented in Table 1 and Table 2.

Table 1. Input parameters for benchmark simulations.

κ^*	λ^*	r_s	υ_{ur}	OCR_v	K_0	K^{nc}_0
0.02	0.2	200	0.15	1.01	0.6	0.57

[1] $K_0CU = K_0$-Consolidated triaxial test with specified vertical and horisontal stress and then sheared undrained.

Table 2. Input for the failure criterion.

φ (deg)	ψ (deg)	c´ (kPa)
30°	0°	0

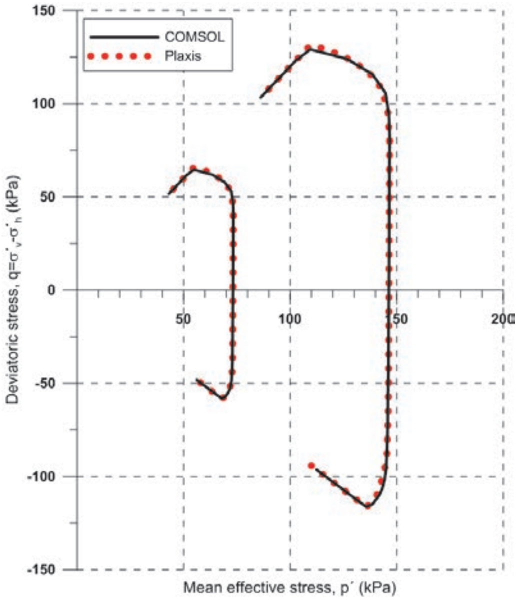

Figure 3. p´-q stress plot comparison between EVP-model in COMSOL and SSC model in Plaxis.

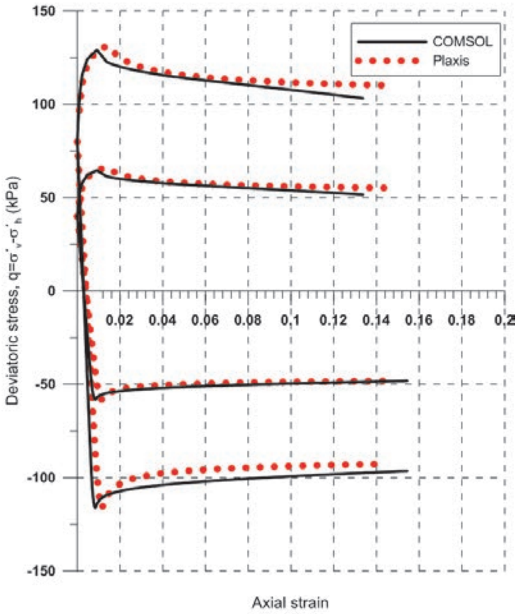

Figure 4. Stress-strain plot comparison between EVP-model in COMSOL and SSC model in Plaxis.

4.1 Discussion

As shown in Figure 3 and Figure 4 the implemented material model shows very similar results to the SSC model in Plaxis, as expected.

The small differences that could be seen in Figure 4 could be one of many reasons e.g. the implementation is not identical, the solver is different etc.

5. Validation of model

In this chapter a comparison with some common laboratory tests will be conducted and some other important features will be shown.

The laboratory tests are conducted on soft clay from a site located just north of Gothenburg, Sweden. The comparison comprises of four K_0CU triaxial tests, one compression and one extension test for clay from 12m and 18m depth respectively, and a constant rate of strain test (CRS) from 24 m depth. The simulations are conducted by using axi-symmetry which reduces the number of nodes of the model tremendously.

The soil samples are modeled with a height of 100 mm for the triaxial test and 20 mm for the CRS test. The diameter is the same for both tests and is 50 mm.

5.1 Comparision with laboratory tests

The evaluated parameters used as input in the EVP model for the triaxial test simulations are summarized in Table 3 and Table 4. The vertical stress was set to 72 kPa and 108 kPa for the 12m and 18 m soil sample respectively.

Table 3. Input parameters for triaxial laboratory simulations.

κ^*	λ^*	r_s	υ_{ur}	OCR_v	K_0	K_{nc}
0.02	0.2	125	0.15	1.36	0.6	0.55

Table 4. Input for the failure criterion.

φ (deg)	ψ (deg)	c´ (kPa)
34°	0°	4

The results from the EVP model compared with the triaxaial tests are presented in Figure 5 and Figure 6.

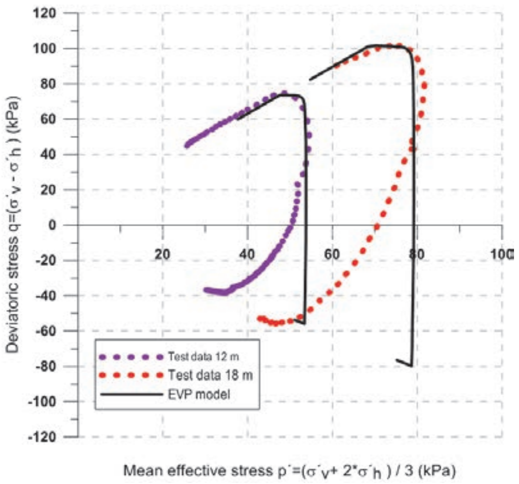

Figure 5. p´-q stress plot comparison between EVP-model in COMSOL and triaxial laboratory tests .

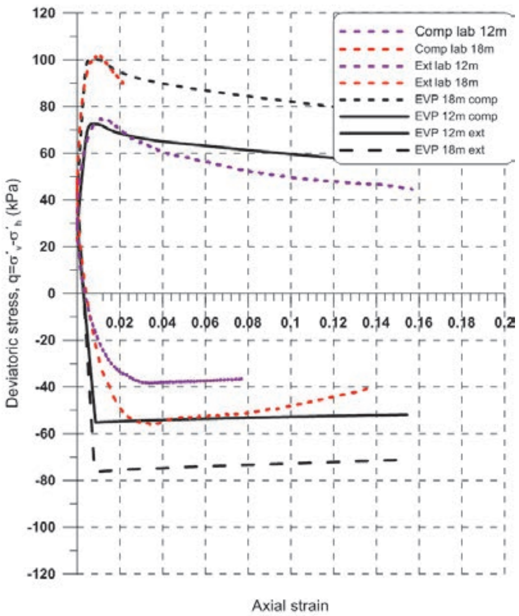

Figure 6. Stress-strain plot comparison between EVP-model in COMSOL and triaxial laboratory tests.

The evaluated parameters used as input in the EVP model for the CRS test simulations are summarized in Table 5 and the comparision between the EVP model and the CRS laboratory test is presented in Figure 7.

Table 5. Input parameters for CRS laboratory simulations.

κ^*	λ^*	r_s	υ_{ur}	OCR_v	K_0	K^{nc}_0
0.015	0.23	130	0.15	1.35	0.6	0.55

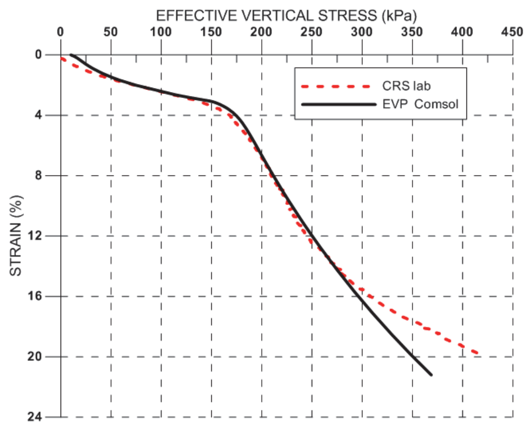

Figure 7. Stress-strain plot comparision between EVP-model in COMSOL and CRS laboratory test from 24 m depth.

5.2 Effect of strain rate

To demonstrate some of the capabilites of the model a K_0CU compression test is conducted with three different strain rates, namely 0.1, 0.01 and 0.001 mm/hr. The input parameters are the same as for the benchmark above with the only difference that the isotropic preconsolidation stress is set to 90 kPa.

The result are presented in Figure 8 and Figure 9.

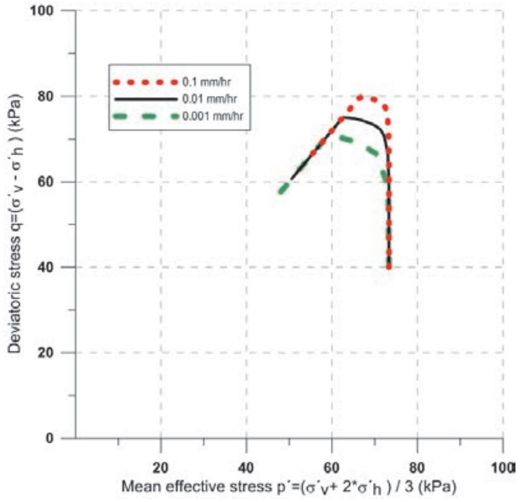

Figure 8. Effect of different strain rates plotted in a p´-q stress plot.

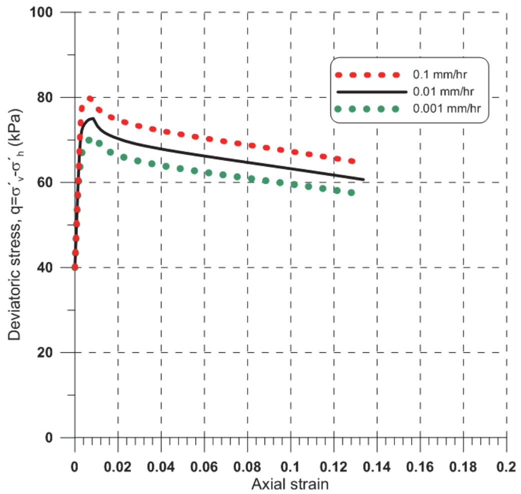

Figure 9. Effect of different strain rates plotted in stress-strain plot for sample of h=100 mm and d=50 mm.

5.3 Discussion

The validation of the model shows good agreement for the triaxial compression tests but a very poor agreement for the triaxial extension tests. The latter is as expected as the model has an isotropic formulation of the reference surface (cap). The same applys to SCC model used in the benchmarking.

For the simulation of the CRS test there is a good agreement to about 14% strain where the simulation starts to diverge from the laboratory test curve. This is also as expected due to the formulation that originates from the Modified Cam-Clay, see e.g. [6].

6. Conclusions

This paper describes the implementation of an isotropic Elastic Visco-Plastic (EVP) material model for soft soils in COMSOL. The benchmark against a very similar material model (SCC) showed good agreement. Even so, one could argue that the EVP model needs to be benchmarked against real tests and not just other material models. The implemtation in COMSOL seems however to be acceptable.

The validation of the model by comparison with laboratory tests showed somewhat varying results. There was a very good agreement for the development of stresses in the compression triaxial test, even if the decrease of the deviatoric stress beyond the peak stress was somewhat underestimated. Also for the CRS test the agreement was very good. The overestimation of strains >14% would be of less importance in real design as such large strains probably would not be acceptable. Even so important features of soft soil behaviour such as strain rates effects can be captured.

However, for the extension triaxial tests the agreement was far from satisfactory. The peak strength was highly overestimated. Nor does the model capture the viscous behavior in the extension tests. But, this is as anticipated.

The applied model has, as said previously, an isotropic formulation of the reference surface, and it is a well-known fact that such models have problems of capturing the behavior of extension tests. Thus, the poor agreement is a result of the draw-backs of the EVP model and not the implementation in COMSOL. As a consequence, future development an anisotropic formulation of the reference cap has to be incorporated in the EVP-model.

Using COMSOL for such a development would be a great advantage as it offers a user friendly interface for adding complexity to a model in a step-by-step manner without the need of any in depth skills of programming language.

7. References

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