Coupling Forced Convection in Air Gaps with Heat and Moisture Transfer inside Constructions

M. Bianchi Janetti[1], F. Ochs[1] and R. Pfluger[1]
[1]University of Innsbruck, Unit for Energy Efficient Buildings, Innsbruck, Austria
*Corresponding author: Technikerstr. 13, A-6020 Innsbruck, email: michele.janetti@uibk.ac.at

Abstract: In this paper a model for coupling heat and moisture transfer inside solid domains with convective transfer through thin air gaps between the solid domains is addressed. As applicative example, the time-dependent moisture and temperature distributions inside the external wall of a building with internal insulation crossed by a wooden ceiling is calculated. An air gap between the beam-end of the wooden ceiling and the wall has been modeled.

Keywords: Heat and moisture transfer, convection, building-physics, internal insulation, beam-end.

1. Introduction

The numerical simulation of humidity and temperature distributions inside constructions is useful for developing the insulation envelope of buildings, selecting appropriate materials and design.

Energy and moisture transfer mechanisms can be described by a system of two partial differential equations derived from the energy and moisture equilibrium balances. In order to capture all the significant transfer mechanisms (heat and vapor diffusion, liquid water transfer due to capillary pressure gradients) the material properties are described by functions of the system variables (temperature and relative humidity) [1], [2].

It is known that air gaps can strongly influence the temperature and moisture distribution inside the construction. Hence, for a realistic simulation it is sometimes necessary to include convective transfer mechanisms inside air gaps between the solid construction components.

However, the fully determination of velocity, temperature and humidity fields inside the air gaps due to computational fluid dynamics and the coupling of these with the solids domains would lead to very high computational effort.

In this paper a simplified model is presented: The model applies for the case of forced convection inside thin air gaps between the constructions components. The convective transfer (moisture and energy) is considered one-dimensional along the gap axis (line source model). The governing equations for convection (1D) are assigned in the weak-form on the boundary of the solid domains and coupled with heat and moisture transfer (2D) inside the construction. In this way the influence of the streaming air in the gap can be quantified with limited computational effort.

2. Problem Description

As applicative example, the temperature and moisture distribution in a wooden ceiling with wooden beams crossing an external wall of a building with internal insulation is simulated.

In the air gap between the beam ends and the wall, air flows due to the pressure difference between the upper and the lower side of the ceiling. The case study is modeled in two dimensions, considering the section through the beam axis as shown in **Figure 1**.

A 3D analysis of a similar problem has been addressed in [3].

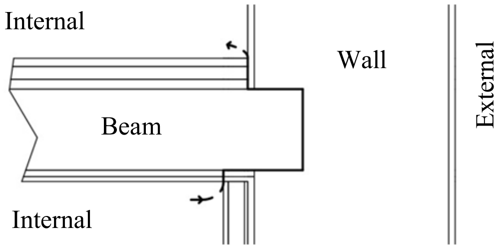

Figure 1. Section view of the ceiling-wall junction. Air is streaming in the gap between beam and wall

3. Governing Equations and Use of COMSOL Multiphysics

The heat and moisture transfer processes in the solid domains (constructions porous materials) can be described by a system of two partial differential equations derived by imposing the equilibrium balance of mass and energy within an infinitesimal element of volume. Following the governing equations are reported in the one dimensional form for the sake of simplicity:

$$\frac{\partial u}{\partial \varphi}\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial x}\left(-D_{m,\varphi}\frac{\partial \varphi}{\partial x} - D_{m,T}\frac{\partial T}{\partial x}\right) = 0 \qquad (1)$$

$$\frac{\partial h}{\partial T}\frac{\partial T}{\partial t} + \frac{\partial h}{\partial \varphi}\frac{\partial \varphi}{\partial t} + \frac{\partial}{\partial x}\left(-D_{e,T}\frac{\partial T}{\partial x} - D_{e,\varphi}\frac{\partial \varphi}{\partial x}\right) = 0 \qquad (2)$$

Temperature T and relative humidity φ are the dependent variables whereas t and x represent time and position. u is the moisture content and h the enthalpy. $D_{m,\varphi}$, $D_{m,T}$, $D_{e,T}$ and $D_{e,\varphi}$ are material specific functions depending from T and φ. The derivation of the transfer functions has been reported in [4]. Equations (**1**) and (**2**) are introduced in Comsol Multiphysics selecting in the Model Navigator: PDE Modes → PDE, Coefficient Form → Time-dependent analysis. The equation coefficients are assigned in the Subdomain Setting-Window of the Comsol Multiphysics GUI.

The forced convection of moisture and energy inside the thin air gap between the wooden beam and the external wall is described by the following balance equations based on the line source model:

$$A\left(\frac{\partial \rho_v}{\partial t} + v\frac{\partial \rho_v}{\partial s}\right) = L\beta_k\left(p_{v,b} - p_{v,a}\right) \qquad (3)$$

$$A\left(\frac{\partial h_a}{\partial t} + v\frac{\partial h_a}{\partial s}\right) = L\alpha_k\left(T_b - T_a\right) \qquad (4)$$

where s and t represent the position along the gap axis and the time. ρ_v and p_v are the water vapor density and the partial pressure of water vapor in the air. h_a and T_a are the specific enthalpy of the air and the air temperature. T_b and $p_{v,b}$ are the temperature and the partial pressure of the water vapor on the boundary of the air gap. β_k and α_k are the convective transfer coefficients for moisture and energy at the gap surfaces. v represent the air velocity, which is supposed to be directed along the gap axis, A *is* the gap cross-section area and L is the length of the cross-section perimeter.

The partial pressure of water vapor depends on the temperature and the water vapor density through the ideal gas equation ($p_v = \rho_v R_v T$).

The specific enthalpy of humid air is described by the following equation:

$$h_a = \rho_a c_{p,a} T_a + \rho_v h_{lv} \qquad (5)$$

where ρ_a and $c_{p,a}$ represent the air density and the air heat capacity. h_{lv} represents the latent heat of evaporation. In first analysis the term ($\rho_v h_{lv}$) is negligible.

Equations (**3**) and (**4**) are implemented in Comsol Multiphysics in the weak form selecting in the Model Navigator: PDE Modes → Weak Form, Boundary → Time-dependent analysis.

In the next step, the internal boundaries which represent the air gap are selected and activated in the window “boundary settings”. The weak formulation of the equations (**3**) and (**4**) is introduced in the field “weak term”.

4. Boundary Conditions, Initial Conditions and Material Properties

On both, the external and internal surfaces boundary conditions of the third kind with constants heat and mass transfer coefficients (α and β) are imposed.

For the sake of simplicity in this work, the time dependent climatic data (temperature $T_e(t)$ and relative humidity $\varphi_e(t)$) are approximated with sinus curves presenting annual period. The influence of the rain and of the radiation is not taken in to account. On the internal surfaces constant values of temperature and relative humidity are imposed (see **Table 1**).

The initial conditions inside the construction are 20 °C and 80 % relative humidity.

Table 1 Boundary conditions

Parameter	Unit	Int.	Ext.
θ	[°C]	20	$T_e(t)$
φ	[%]	50	$\varphi_e(t)$
α	[W/(m^2 K)]	6	25
β	[s/m]	3e-08	2e-06

On the internal boundaries between the beam-end and the wall, the following source terms are assigned:

$$j = -L\beta_k(p_{v,b} - p_{v,a}) \tag{6}$$

$$\dot{q} = -L\alpha_k(T_b - T_a) \tag{7}$$

where j and q represent the vapour and the heat flux exchanged between the wet air streaming through the air gap and the solid domains.

The determination of the convective coefficients for heat transfer (α_k) and for the vapour transfer (β_k) present some difficulties since the real shape of the gap and the flow regime of the air inside are not known. Here an infinite plane gap and laminar flow are considered. Moreover, the transfer coefficients are assumed to be constant all over the gap length and have been assessed using correlations reported in [5] (see **Table 2**).

Table 2 Parameters for the calculation of convective transfer in the air gap

Parameter		Unit	Value
Gap thickness	d	[m]	0.002
Max. air velocity	v_{max}	[m/s]	1.25
Mass transfer coefficient	β_k	[s/m]	3e-7
Heat transfer coefficient	α_k	[W/(m^2 K)]	46.2

For the velocity of the air stream through the gap a simplified artificial profile has been chosen (see Figure **2**) in order to be able to clearly distinguish between the different effects. In reality the air steam is driven by the stack effect (pressure difference between different floors due to density difference).

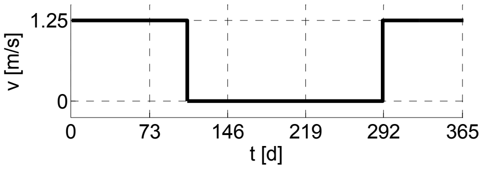

Figure 2 Velocity of the air stream through the gap

5. Solver Settings and Spatial Discretization

Standard solver parameters are used with the exception that the maximum time step is limited to 3600 s.

The spatial discretisation grid is composed by 1008 quadrilateral elements (Lagrange-quadratic elements, see **Figure 3**).

The model presents 8116 degrees of freedom. The solution for the simulation of one year has been obtained after about 20 minutes calculation time (processor: intel(R) Core(TM) i7 CPU M 620 @ 2.67GHz, one core, RAM: 4.00GB)

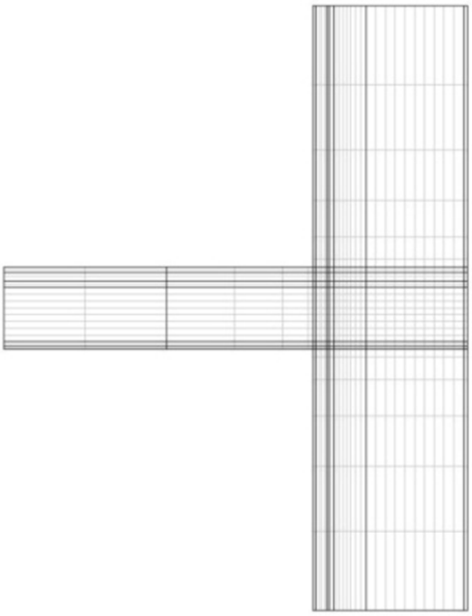

Figure 3. Beam-end model - spatial discretization

6. Results

The sensitivity of the model was investigated concerning three input-parameters (v_{max}, α_k and β_k). The simulated variants are compared in **Table 3**. The case 0 represents the reference case (the input parameters assume the values shown in **Table 2**). For this reference case the water activity distribution is shown for three different time steps in **Figure 4**, **Figure 5** and **Figure 6** (note that the relative humidity inside the construction is denominated water activity, a_w-value). Consider that these three time steps refer to the period between October and December, since the simulation starts at the 1th of January (external winter conditions).

The effect of the convection in the gap is remarkable especially at the upper corner of the beam-end, where the a_w-value increase up to 0.8 at the end of the year (**Figure 6** and **Figure 9**).

In **Figure 7**, **Figure 8** and **Figure 9** the water activity trend at three different points (see **Figure 6**) is reported and compared with the state without convection.

It can be observed that in all the considered cases the air streaming dry the construction at position 1, whereas in some cases it can lead to increasing a_w-value at position 2 and 3 during the winter period.

The velocity of the streaming air in the gap has been varied, showing that this parameter influences significantly the simulation results. Moreover, it has been verified that the heat transfer coefficient and the convective transfer coefficient have a small influence on the simulation results, at least within the investigated range of variation (cases 2 and 3).

In order to quantify the influence of the three considered input parameters on the simulation results, the deviation (mean and max. values over the time) with respect to the reference case of the a_w-value at point 3 (see **Figure 6**) is reported in **Table 4**.

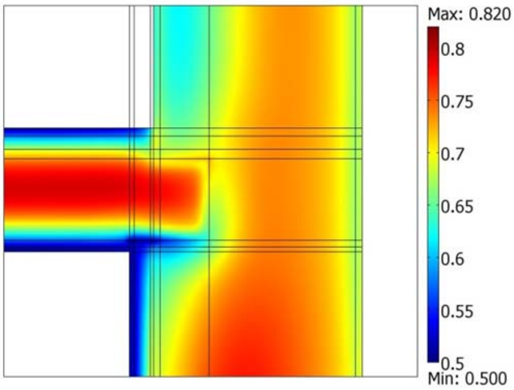

Figure 4. Moisture distribution (a_w-value) after 325 days. Case 0 (see **Table 3**)

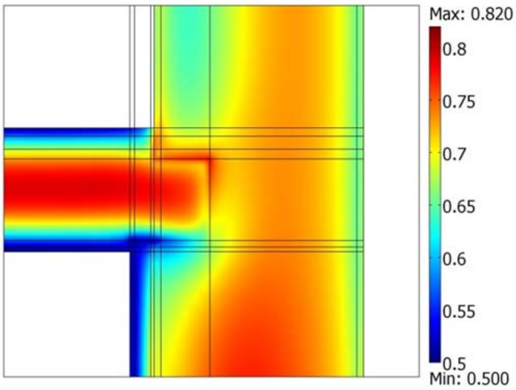

Figure 5. Moisture distribution (a_w-value) after 345 days. Case 0 (see **Table 3**)

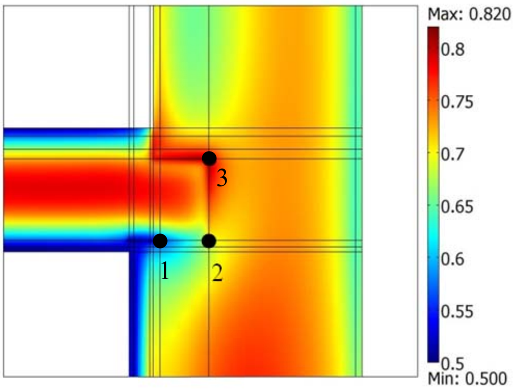

Figure 6. Moisture distribution (a_w-value) after 365 days. Case 0 (see **Table 3**). The positions 1, 2 and 3 refer to **Figure 7**, **Figure 8** and **Figure 9**

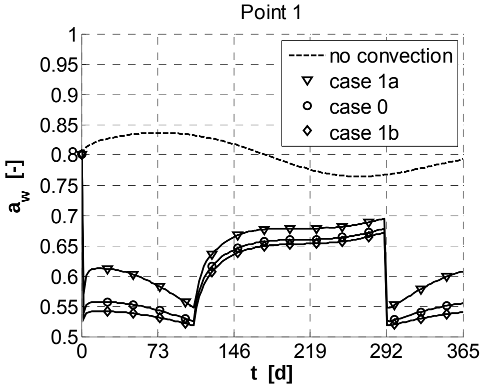

Figure 7. Development of water activity a_w at point 1 for different velocities of the air stream through the gap (see **Figure 6**)

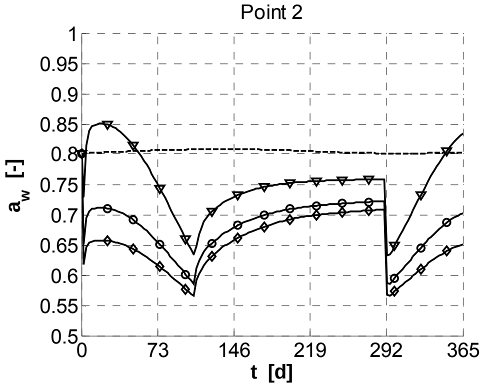

Figure 8. Development of water activity a_w at point 2 for different velocities of the air stream through the gap (see **Figure 6**)

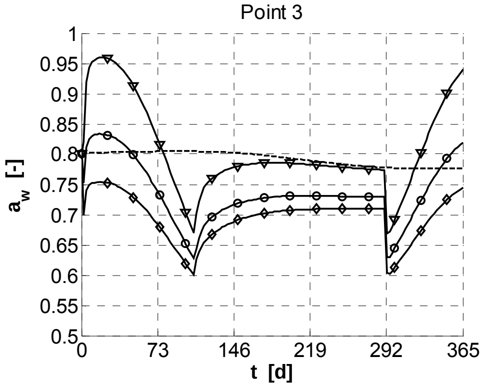

Figure 9. Development of water activity a_w at point 3 for different velocities of the air stream through the gap (see **Figure 6**)

Table 3 Simulated variants

Case	**Var. v_{max}**	**Var. α_k**	**Var. β_k**
0	-	-	-
1a	-50%	-	-
1b	+50%	-	-
2a	-	-50%	-
2b	-	+50%	-
3a	-	-	-50%
3b	-	-	+50%

Table 4 Deviation with respect to the reference case (case 0) of the a_w-value at point 3 (**Figure 6**). Mean and max. values over the time.

a_w deviation - mean value [%]			
Case	**Var. v_{max}**	**Var. α_k**	**Var. β_k**
a	7.2044	-0.1608	0.0605
b	-4.0241	0.1907	-0.0057

a_w deviation - max. value [%]			
Case	**Var. v_{max}**	**Var. α_k**	**Var. β_k**
a	12.685	0.7077	0.4416
b	-7.9409	1.3958	-0.0139

7. Conclusions and Outlook

Using the line source model described in this work, the air streaming through thin gaps can be taken in to account when calculating the moisture distribution inside constructions.

The sensitivity of the model concerning three input parameters (v_{max}, α_k and β_k) is investigated.

Further work should concern the determination of α_k and β_k which for the sake of simplicity are considered constant in this study.

Moreover, for a realistic calculation of the moisture distribution, as boundary conditions realistic climatic data including driving rain and radiation should be included.

The pressure drop of the gap will be taken from laboratory measurements.

8. Nomenclature

A	$[m^2]$	*Cross-section area*
c	$[J/(kg\ K)]$	*Heat capacity*
Dm,φ	$[kg/(m\ s)]$	*Transport Coefficients*
Dm,T	$[kg/(m\ s\ K)]$	
De,φ	$[W/m]$	
De,T	$[W/(m\ K)]$	
h	$[J/m^3]$	*Volumetric enthalpy*
j	$[kg/(m\ s)]$	*Mass source*
L	$[m]$	*Cross-section perimeter*
p	$[Pa]$	*Pressure*
q	$[W/m]$	*Heat source*
R	$[J/(kg\ K)]$	*Gas constant*
t	$[s]$	*Time*
T	$[K]$	*Temperature*
θ	$[°C]$	*Temperature*
u	$[kg/m^3]$	*Volumetric water content*
α	$[W/(m^2K)]$	*Heat transfer coefficient*
β	$[s/m]$	*Mass transfer coefficient*
ρ	$[kg/m^3]$	*Density*
φ	$[\%]$	*Relative humidity*
a_w	$[-]$	*Water activity*

Subscripts

a	*Air*
b	*Boundary*
e	*External*
k	*Convective*
l	*Liquid*
p	*Constant pressure*
v	*Vapor*

9. References

[1] H. Künzel and K. Kiessl, “Calculation of heat and moisture transfer in exposed building components,” *International Journal of Heat and Mass Transfer*, vol. 40, no. 1, pp. 159–167, Oct. 1996.

[2] J. Grunewald, “Diffusiver und konvektiver Stoff- und Energietransport in kapillarporösen Baustoffen,” *Dissertation, Technischen Universität Dresden*, 1997.

[3] M. B. Janetti, F. Ochs, R. Pfluger, and W. Feist, “HYGROTHERMISCHE 3D SIMULATION VON BAUTEILEN MIT COMSOL MULTIPHYSICS,” *BAUSIM Berlin (accepted)*, 2012.

[4] M. Bianchi Janetti, F. Ochs, and W. Feist, “3D Simulation of Heat and Moisture Diffusion in Constructions,” *Comsol Conference Stuttgart 2011*, 2011.

[5] H. D. Baehr and K. Stephan, *Wärme- und Stoffübertragung*, 6. Auflage. 2008.