Estimation Of Hydraulic Conductivity For An Heterogeneous Unsaturated Soil Using Electrical Resistivity And Level-Set Methods

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Abstract: The estimation of the soil saturated hydraulic conductivity (*Ks*) is crucial in understanding water flow and transport of contaminants. There are many hydrological techniques available in helping to determine this parameter. In the framework of a PhD project, an iterative scheme for estimating Ks in an unsaturated medium was developed using infiltration test, time-lapse ERT and level-set method. This is accomplished by incorporating geophysical, hydrological and imaging methods into the proposed technique. In this study, 1D and 2D hydrology models, and 2D resistivity model were used to validate the technique.

Keywords: electrical resistivity method, level-set method, hydrology, inversion, unsaturated heterogeneous medium

1. Introduction

The determination of the subsurface saturated hydraulic conductivity (*Ks*) provides valuable information regarding the hydro-geological characteristics of the groundwater flow and a better determination in the potential displacement of groundwater or contaminants. Constant head method, in-situ soil analysis, empirical and semi-empirical hydro-geophysical relations, and other methods, can be used in determining *Ks*. While these techniques can provide quality data points, they are often limited by sparse data sampling and scale, low survey coverage and mostly restricted to particular sites. In order to address these problems and limitation, an iterative scheme is developed for estimating the conductivity *Ks* with the use of electrical resistivity method (ERT), level-set method and hydrology method (water infiltration). COMSOL Multiphysics is used to solve forward solutions for the electrical and hydrological models.

2. Theory (Iterative Scheme)

The determination of the saturated hydraulic conductivity (K_S) is done by an iterative scheme that estimate K_S by tracking the velocity of the infiltration front or flow front (equation 1). This scheme minimizes the difference in distance travelled (or velocity) between the measured and the modeled flow front (equation 2). *Ks* is said to have converged when $\Delta V = 0$. The van Genuchten water retention model is used to describe the physics of the soil in the forward modeling process (van Genuchten, 1980). The spatial variation in the van Genuchten parameters (α, n, l) is assumed to be small therefore they are kept constant throughout the forward and the inverse process. These parameters can be determined using time-domain reflectometry (*TDR*) during the ERT monitoring survey.

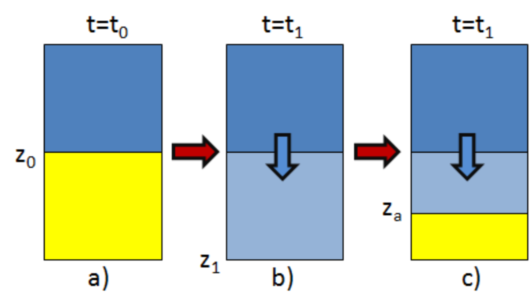

Figure 1. Process flow in estimating the saturated hydraulic conductivity. Measured water infiltration front at time a) t_0 and b) t_1. c) Modeled water infiltration front at time t_2 using the estimated K_i.

$$K_{i+1} = K_i + \Delta V \tag{1}$$

$$\Delta V = V_{measured} - V_{modeled} \tag{2}$$

$$V_{measured} = \frac{\Delta z}{\Delta t} = \frac{z_1 - z_0}{t_1 - t_0} = K_1 \tag{3}$$

$$V_{modeled} = \frac{\Delta z_a}{\Delta t} = \frac{z_1 - z_a}{t_1 - t_0} \tag{4}$$

3. Methodology

The proposed *Ks* estimation scheme can be separated into three parts: hydro-geophysical method, combined hydrology and level-set methods, and hydro-geological modeling.

3.1 Hydro-Geophysical Method

Controlled water infiltration is done over the survey zone and monitoring electrical resistivity data are obtained. The positioning of the flow front can be delimited in area where the difference in change in resistivity is greater than a pre-determined value. The positioning of the flow front at all time is defined by,

$$\vec{\Gamma} = [\gamma_1, \dots \gamma_t, \dots \gamma_n] \qquad (5)$$

where γ_t is the position of the flow front at time t and n is the total number of time-lapse measurements.

3.2 Hydrology and Level-Set Methods

Level-set method (LSM) is used to reconstruct flow front movement between monitoring time (Osher and Sethian, 1988). The LSM propagation equation is defined by equation 6.

$$\frac{\mathrm{d}\phi_t}{\mathrm{dt}} + F(\phi_t, t)|\nabla\phi_t| = 0 \qquad (6)$$

where $F(\phi_t, t)$ is the velocity function of the level-set and ϕ_t correspond to the distance to the closest point of the boundary γ_t. The simplest velocity function that can be defined is to assume that the infiltration front moves at a speed proportional to its distance between 2 consecutive times (T_i and T_{i+1}).

Once the interpolated flow fronts are obtained using LSM at various discrete times, hydraulic channels are determined by placing initial markers at the initial flow front and by assuming that the channels propagate normal to the flow fronts. The distance travelled by the hydraulic channels can be calculated and the flow front velocity can be determined and expressed as the following vector equation,

$$\vec{\mathrm{k}} = [k_1, \dots k_i, \dots k_n] \qquad (7)$$

where k_i is the estimated hydraulic conductivity for the hydraulic channel i.

3.2 Hydro-Geological Modeling

In a porous medium, the hydraulic conductivity is defined by equation 8.

$$\mathrm{K}(\theta) = \mathrm{K_S}\,\mathrm{K_r}(\theta) \qquad (8)$$

where K_S is the saturated conductivity, K_r is the relative conductivity and $K(\theta)$ is the conductivity of the medium as a function of pressure head. In order to calculate K_r, van Genuchten retention model is used.

The propagating velocity of the flow front can never surpass the saturated conductivity of the medium. By supposing that the estimate hydraulic conductivity $\vec{\mathrm{k}}$ is the saturated conductivity $\overrightarrow{\mathrm{K_s}}$, the forward hydrology model is solved and the modeled flow fronts are extracted at monitoring times. The modeled flow front vector is defined by equation 9.

$$\vec{\tau} = [\lambda_1, \dots \lambda_t, \dots \lambda_n] \qquad (9)$$

where $\lambda_t = f(x, y)$ is the modeled infiltration front at time t. The velocity of the modeled water infiltration front is calculated and defined as $\overrightarrow{\upsilon_\mathrm{m}}$. The velocity difference between the modeled front and the measured front is calculated and added to the estimated saturated conductivity $\overrightarrow{\mathrm{K_m}}$. The new saturated conductivity is therefore defined by equation 10.

$$\overrightarrow{\mathrm{K_{m+1}}} = \overrightarrow{\mathrm{K_m}} + \left(\vec{\mathrm{k}} - \overrightarrow{\upsilon_\mathrm{m}}\right) \qquad (10)$$

The solution is said to have converged when the modeled front velocity is equal to the measured front velocity. Other criteria can be used such as the difference in front velocity ($\Delta\upsilon$) is less than a certain value β set by the user.

$$\Delta\upsilon = \left|\vec{\mathrm{k}} - \overrightarrow{\upsilon_\mathrm{m}}\right| < \beta \qquad (11)$$

4. Tests, Results and Discussions

4.1 1D Hydrology Modeling

In this section, validation of the iterative scheme is done using 1D hydrology model. A medium consists entirely of loamy sand is used. The van Genuchten parameters are taken from Wosten et al. (2001). A infiltration rate of 1000 kg/(m^2·s) of water is used. The water table is located 7 m below the surface. The surface of the model is located at z = 0 m, therefore hydraulic charge for the boundary condition and the initial condition are set to -7 m. The problem is also speed up by setting K_S in the order of m/s and the measurement time set in the order of seconds.

Table 1: van Genucthen parameters.

Soil Type	Loamy Sand
Ks (m/s)	1.79E-06
Ks_simulated (m/s)	1.785
$\theta_{RESIDUAL}$	0.02
$\theta_{SATURATED}$	0.46
α (m^{-1})	1.44
n	1.534
l	-0.215

Soil water saturation level is measured at time T = 0 and 4 sec., and the position of the infiltration front at T = 4 sec. is determined (figure 2). The iterative scheme has proven successful in estimating K_S with a mean absolute percent error (MAPE) of 1.2 % after 150 iterations (figure 3).

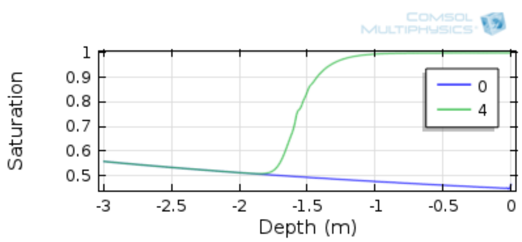

Figure 2. Simulated water infiltration in unsaturated loamy sand with saturation level shown at 0 and 4 sec.

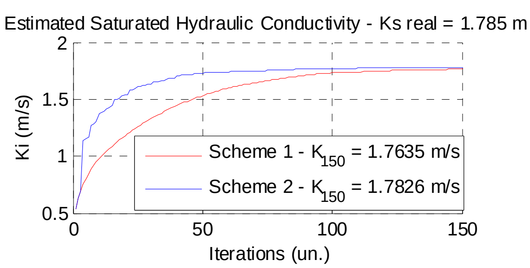

Figure 3. Estimated hydraulic conductivity after 150 iterations using scheme 1 (iterative) and scheme 2 (Weibull + iterative).

It was found that the convergence of K_i as a function of iteration can be fitted by a type 2 sigmoidal Weibull function that is defined by equation 12 (Seber and Wild, 1989).

$$y = A - (A - B)e^{-(Cx)^D} \qquad (12)$$

where A, B, C and D are unknown variables, x is the iteration i, and y is the estimated K_i at iteration i. The Weibull function is solved using the Nonlinear Least Squares method. By integrating the Weibull function into the iterative scheme, the convergence of the problem is speed up. Statistic on the improvement can be seen in table 2.

Table 2: Comparison between the standard iterative scheme and the type 2 sigmoidal Weibull iterative scheme.

Scheme	Iterations	Ks (m/s)	MAPE
Iterative	150	1.7635	1.20%
Weibull Type 2	92	1.7639	1.18%
Weibull Type 2	150	1.7826	0.13%

4.2 2D Hydrology Modeling

In this section, validation of the iterative scheme using the level-set method is done using a simple 2D hydrology model. The hydrology modeling is done using COMSOL Multiphysics with Richard's equation in Porous media and Subsurface flow. In this example, the synthetic model consists of a medium separated vertically by a subsoil material and sand (figure 4). This model may simulate the backfilling of sand between a concrete foundation wall and the natural unexcavated subsoil.

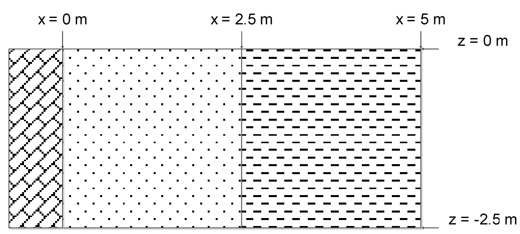

Figure 4. Synthetic model of a concrete impermeable wall (left), with backfilled sand (middle) and the untouched subsoil (right).

The water infiltration rate is 1000 kg/(m^2·s) along the surface and the water table is located at 10 m below the surface (hydraulic head = -10 m).

Table 3: van Genuchten properties for 2D model

Soil Type	Sand	Subsoil
Ks (m/s)	1E-05	1E-06
Ks_simulated (m/s)	10	1
$\theta_{RESIDUAL}$	0.01	0.01
$\theta_{SATURATED}$	0.5	0.5
α (m^{-1})	1	1
n	2	2
l	0.5	0.5

The concrete is taken as an impermeable object and the water infiltration is supposed to have extended beyond the length of the survey. Therefore the supposition that no flow occurs along the x-direction along the vertical boundaries is done. The problem is also again speed up by setting K_S in the order of m/s.

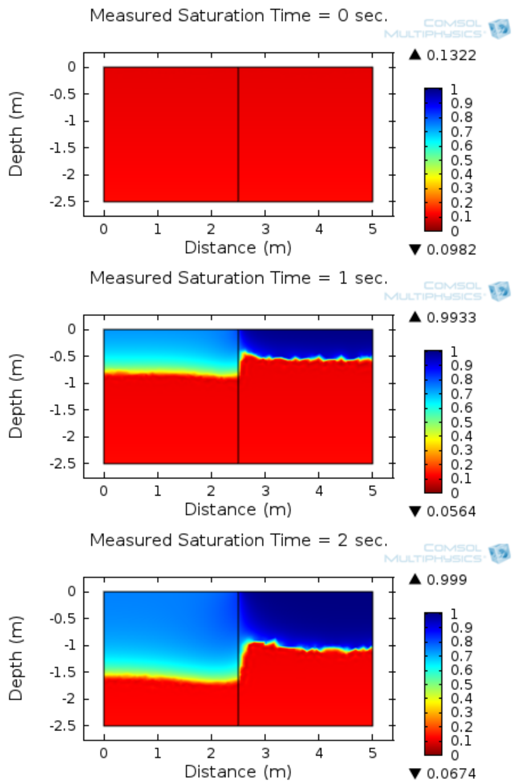

Figure 5. Saturation models at sequential times of 0, 1, 2 and 2.5 sec.

Using the saturation models, the flow front can be delimited by a saturation cut-off of 0.1 in (figure 5). The Level-set method is then used to reconstruct the flow front between sequential times (figure 6).

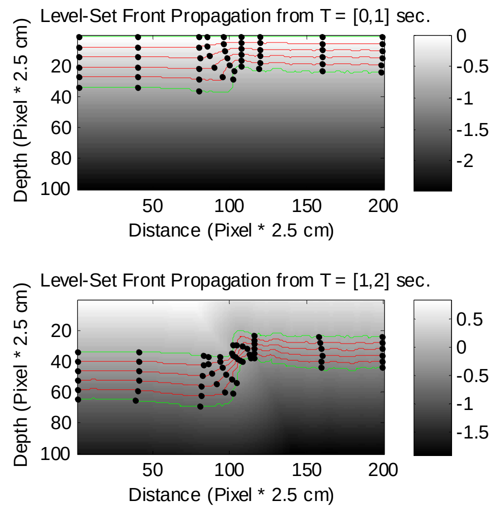

Figure 6. Flow fronts interpolation using LSM.

Water channels are constructed by assuming a flow in the direction that is normal propagation of the flow fronts. Once the channels are constructed, the distance traveled can be calculated and the velocity of the flow can be determined. Using the proposed iterative scheme, the saturation level, the flow fronts and the hydraulic conductivity *Ks* are reconstructed (figure 7). For the saturation level, there is a mean absolute percent error of 1.92 % and 2.29 % for at T = 1 and 2 sec respectively. Regions where Ks is poorly estimated are located in the area where the vertical contact between the sand and the subsoil should be (figure 8). This is in direct result of the estimated flow lines that suppose normal propagation between the flow fronts created by the LSM. This error can be reduced if the time difference between monitoring time is much smaller than the water infiltration.

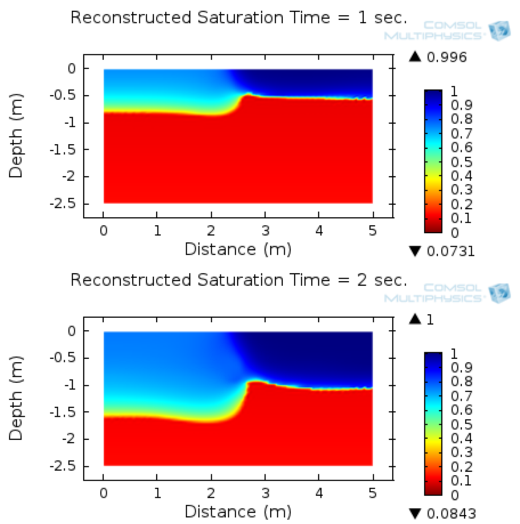

Figure 7. Reconstructed water saturation and flow fronts at time = 1 and 2 sec.

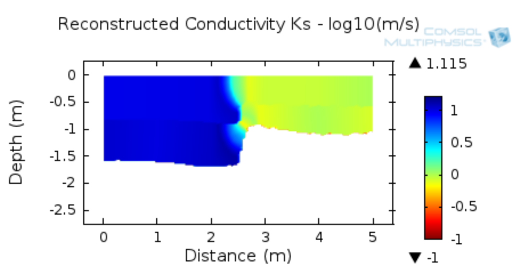

Figure 8. Interpreted hydraulic conductivity model Ks.

4.1 2D Electrical Resistivity Modeling

In this final section, validation of the iterative scheme and level-set method is done using 2D ERT model. Electrical models are obtained by applying Archie's law (1942) to the previous 2D hydrology saturation model (figure 9).

$$\sigma = \sigma_w a^{-1} \theta_s{}^m S_w{}^n \qquad (13)$$

ERT modeling is done using COMSOL Multiphysics with AC/DC module (figure 10). Dipole-dipole array is used for inverting the ERT data. Histograms of resistivity were constructed to determine the position of the flow front (figure 11). Once the flow front is located, the same iterative scheme, as in previous section 4.2, is used.

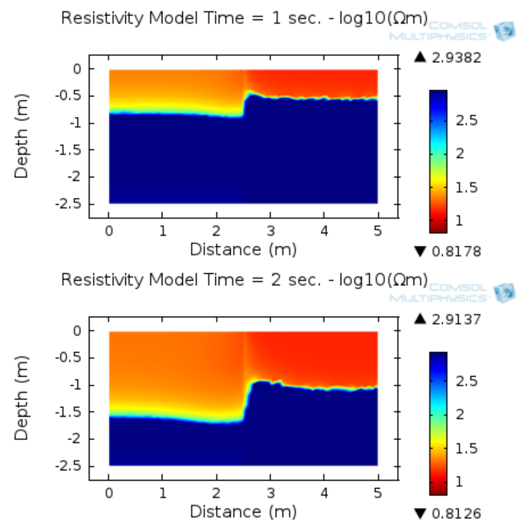

Figure 9. ERT resistivity models.

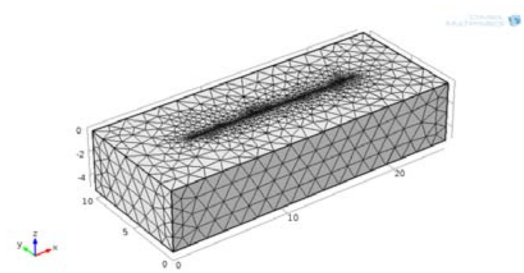

Figure 10. COMSOL resistivity model with meshing and electrodes.

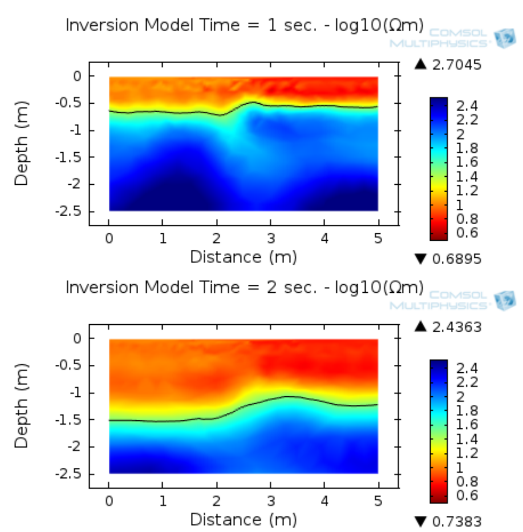

Figure 11. ERT Inversion models. Flow fronts denoted by a black contour line.

Since the exact positioning of the flow front cannot be determined accurately without a priori information, this lack of accuracy will influence the quality of estimation in saturation (figure 12) and in conductivity *Ks* (figure 13). This is a limitation in the vertical and horizontal resolution of the geophysical method. However, the flow fronts determined through ERT inversion models at both monitoring time, are well reconstructed.

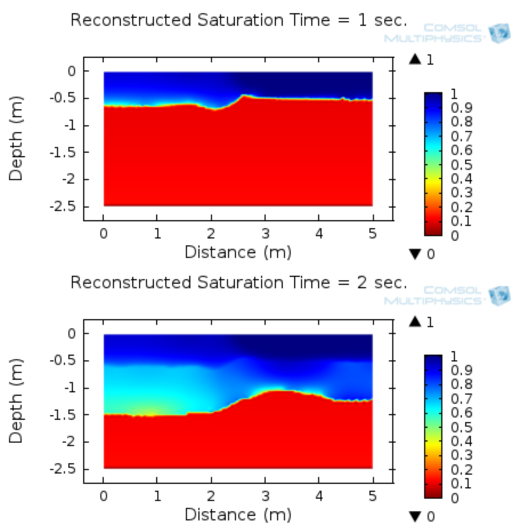

Figure 12. Reconstructed water saturation and flow fronts at time = 1 and 2 sec.

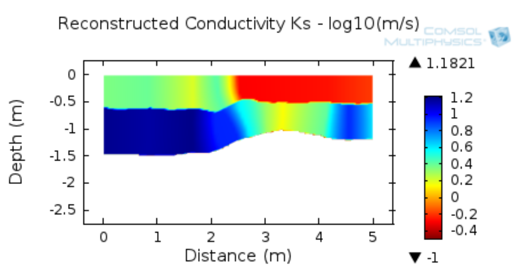

Figure 13. Reconstructed hydraulic conductivity model Ks.

5. Conclusion

We have demonstrated the potential and the capabilities in using electrical tomography measurements in estimating the location of water infiltration front and in using the level-set method in helping to determine the flow line. We have also validated the proposed iterative scheme that estimates the saturated hydraulic conductivity of the medium. Tests have shown a positive reconstruction of the saturated hydraulic conductivity. The limitation of the proposed methodology is determined by several factors:

1. Geophysical ERT method tends to smooth out the model. The vertical resolution and horizontal resolution of an electrical resistivity tomography survey depends on the type of configuration and on the currents and potentials laws. Ground model can also be influenced from lateral effects, artifacts, etc... The depth of investigation is also dependent on the electrodes configuration.

2. The length of the elapsed time between monitoring time ERT measurements plays an important role in the accuracy of flow front reconstruction using the level-set method.

3. If the elapsed time is shorter than the velocity of infiltration, then generally it will produce a better estimation of flow line leading to a better estimation of the saturated hydraulic conductivity.

4. If the elapsed time is longer than the velocity of infiltration, then generally it will produce a poorer estimation of flow lines leading to a more inaccurate estimation of the saturated hydraulic conductivity. This is caused by the fact that the interpolated flow fronts will be largely estimated by the level-set method and it will produce a saturated hydraulic conductivity that consists an average of the different conductivities that the flow front has traversed between monitoring times. This is less true if the soil is more homogeneous than heterogeneous.

6. References

1. van Genuchten, M.Th., A Close-Form Equation For Predicting The Hydraulic Conductivity Of Unsaturated Soils, Soil Science Society of American Journal, **44**, 892-898 (1980)
2. Osher, S., and Sethian, J.A., Fronts propagating with curvature-dependent speed: algorithms based on Hamilton-Jacobi formulations, Journal of Computational Physics, **79(1)**, 12-49, (1988)
3. Wosten, J.H.M., Veerman, G. J., de Groot, W. J. M. and Stolte, J., Waterretentie- en doorlatendheidskarakteristieken van boven en ondergronden in Nederland: De Staringreeks, **153**, (2001)
4. Seber, G. A. F. and Wild, C. J., Nonlinear Regression, John Wiley & Sons Inc., 338 - 339, (1989)
5. Archie, G.E., The electrical resistivity log as an aid in determining some reservoir characteristics, American Institute of Mineral and MEtal Engineering, Technical Publication, **1442**, (1942)