Investigation of Mean-flow Effects on Tubular Combustion Chamber Thermoacoustics Using a Burner Transfer Matrix Approach

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Abstract: The present paper presents a methodology to account for local mean-flow effects on thermo-acoustic instabilities to improve typical thermo-acoustic calculations generally performed under the zero-Mach number assumption. A 3D FEM model of a simplified combustor is solved in COMSOL Multiphysics with the pressure acoustics module. The Helmholtz equation is used to model the combustor and the classical k-τ model for the Flame Transfer Function (FTF) is adopted. In order to account for local non-zero Mach number effects in the burner region, the burner is replaced with its transfer matrix (BTM), computed through the aero-acoustics module considering an assigned mean-flow, and which implicitly takes into account the mentioned effects. The obtained matrix is inserted in the FEM model of the simplified combustor. The BTM ability to represent local mean-flow effects and the impact on the resonant frequencies and their growth rate is then evaluated comparing the results with those provided by an in-house 1D code solving the linearized Navier-Stokes equations in the presence of a mean flow.

Keywords: Thermoacoustic instabilities, mean flow, burner transfer matrix.

1. Introduction

Lean premixed combustion technology can be considered the most effective solution to meet more and more stringent regulations on pollutant emissions, in particular NOx, of these last years. One of the most critical issues of lean combustion technology is the occurrence of combustion instabilities related to a coupling between pressure oscillations and thermal fluctuations excited by unsteady heat release [1]. Such instabilities may compromise the combustor life and integrity so that the prediction of the thermoacoustic behaviour of the system becomes of primary importance.

Several methods are used to predict the thermoacoustic instabilities of a combustor. The full three-dimensional unsteady Navier-Stokes equations might be solved as in Large Eddy Simulations. Several works have demonstrated its capabilities to represent the flame dynamic and its interaction with acoustic waves. On the other hand, it does not provide any insight on why and how to control instabilities [2]. Moreover, it remains extremely CPU demanding.

In low-order methods the geometry of the combustor is modelled by a network of homogeneous (constant density) 1D acoustic elements where the acoustic problem can be solved analytically [2]. The main drawback is that the geometrical details of the combustor cannot be accounted for and, due to the (quasi)-1D assumption, only the first longitudinal modes, below the cut-off frequency of the element are solved.

Finite Element Methods (FEM) may be used to solve for the complete 3D problem. The set of linear transport equations for the perturbations of velocity, temperature and density can be derived by linearizing the Navier-Stokes equations [3], where the local unsteady heat release appears as a forcing term. It is often assumed that the mean flow is at rest so that a wave equation for the acoustic perturbations can be derived.

In some regions of gas turbine and aero-engine combustors, such as the burner region, the Mach number is not negligible and assuming the mean flow at rest may leads to errors in the stability prediction of principal modes [2].

In the present work a methodology to account for local mean-flow effects on thermoacoustic instabilities in zero-Mach number calculation is presented. A 3D FEM model of a simplified combustor is solved in COMSOL replacing the burner with its transfer matrix. This is computed with COMSOL aero-acoustics module imposing a mean-flow, reproducing numerically the experimental two-source technique. The BTM ability to represent local mean-flow effects and the impact on the resonant frequencies and their growth rate is then evaluated and results are compared with those provided by an in-house 1D code solving the linearized Navier-Stokes equations in the presence of a mean-flow.

2. Mean Flow

In gas turbine combustion systems a mean flow is generally present which brings fresh reactants in the combustion zone [4]. Typically, the Mach number in the plenum and flame tube regions are small (less than 0.1) while they assume not negligible values in the burner region, connecting the formers. A common assumption is to neglect mean flow leading to a simplification of the governing equations and, at the same time, introducing errors which may have a severe influence on the stability prediction of the principal modes.

In general, three types of fluctuations are coupled in a combustor: acoustic waves, entropy waves and vorticity waves [5]. Acoustic perturbations propagate at the speed of sound augmented by the local mean velocity while the last two modes are simply convected by the mean flow. The main consequence of assuming a zero-Mach number is that neither the entropy nor the vorticity mode can propagate so that a wave equation for the acoustic perturbations can be derived. That is, both dissipation and the effect on acoustic propagation are neglected.

Entropy is generated unsteadily in the flame region (hot spots) and convected by the mean flow, altering the fluctuating fields. It has been observed [4] to be of importance only for the lowest-frequency modes. This is mainly due to the fact that the high-frequencies entropy disturbances are smoothed out by turbulent mixing and diffusion as they are convected downstream so that they may be negligible by the time the wave reaches the combustor exit [4]. Once the entropy wave interacts with the boundary it may be partially converted into acoustics [6] and it may enhance or damp the thermoacoustic modes of the system.

The approach proposed in the present paper adopts a 3D FEM, solving the wave equation for the propagation of the only acoustic wave under the zero mean flow assumption, for the plenum and combustor regions. The burner region, where Mach number effects are non-negligible, is modelled through a transfer matrix approach, computing the BTM in the presence of a mean-flow, introducing local dissipation effects in the acoustic simulation.

3. Mathematical Model

As already pointed out, under certain hypotheses, i.e. negligible mean flow velocity, absence of viscous losses and heat conduction and fluid treated as an ideal gas with constant specific heat ratio, the mathematical model that is generally used to describe the acoustic problem in combustors is the following inhomogeneous wave equation [6]:

$$\frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\gamma - 1}{c^2}\frac{\partial q'}{\partial t} \qquad \text{Eq. 1}$$

where q' is the fluctuation of the heat input per unit volume, p is the pressure and c the sound velocity. With the prime it is indicated a perturbation over the time averaged mean value and with the over bar a mean value is denoted.

The solution consists in determining the resonant frequencies of the combustor and the stability of the modes associated to them [1].

It is possible to write the generic fluctuating quantity ϕ' as:

$$\phi'(t) = Re(\hat{\phi}\, exp(i\omega t)) \qquad \text{Eq. 2}$$

ω is a complex quantity whose real part represents the frequency of oscillations and the imaginary one the growth rate of oscillations which characterize the stability of a mode. An unstable mode will have a negative imaginary part meaning an amplitude of the fluctuation growing with time.

Substituting, the Eq. 1 becomes the inhomogeneous Helmholtz equation:

$$\frac{\lambda^2}{c^2}\hat{p} - \nabla^2\hat{p} = -\frac{\gamma - 1}{c^2}\lambda\hat{q} \qquad \text{Eq. 3}$$

The quadratic eigenvalue ($\lambda = -i\omega$) problem here above is solved with the acoustic module of COMSOL Multiphysics by means of an iterative linearization procedure.

The heat release fluctuations $\hat{q}$ are generally related to the acoustic fluctuations at the injection location through a so-called Flame Transfer Function (FTF). A widely used formulation for the FTF that is adopted in this work is the following:

$$\frac{\hat{q}}{\bar{q}} = -k\frac{\hat{u}_i}{\bar{u}_i}exp(-i\omega\tau_{conv}) \qquad \text{Eq. 4}$$

where $\bar{u}_i$ is the air mean velocity at the fuel injection location, $\bar{q}$ is the mean heat release rate per unit volume, k the index while τ_{conv} the time delay which is generally computed considering the convection time from injection plane to the flame location.

3.1 Burner Transfer Matrix

An element inside of an acoustic system can be replaced by its transfer matrix (TM) linking the acoustic variables at the upstream and downstream sections (see Eq. 5).

$$\begin{bmatrix} \frac{p'}{\rho c} \\ u' \end{bmatrix}_d = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{bmatrix} \frac{p'}{\rho c} \\ u' \end{bmatrix}_u \qquad \text{Eq. 5}$$

The BTM approach involves determining four complex values (T_{ij}) from numerical simulations or experiments. Otherwise, a theoretical formulation for the BTM can be retrieved from the application of the conservation equations.

As already said, in the present work the BTM is numerically computed, under the hypothesis that the wave has a planar behaviour at the element interfaces, using the aero-acoustics module in COMSOL, as described in detail in section 4.2. The resulting TM carries the information on the effects of the mean flow (e.g. dissipation effects). A theoretically derived formulation for the BTM is also tested. The formulation has been proposed by Fanaca [7] and Alemela [8] for one-dimensional flow with low Mach number within a “compact element”, variable cross section and pressure losses. The final expression of the TM is reported below:

$$\begin{bmatrix} \frac{p'}{\rho c} \\ u' \end{bmatrix}_d = \begin{pmatrix} 1 & M_u - \alpha M_d(1+\xi) - ikl_{eff} \\ \alpha M_u - M_d & \alpha + M_d ikl_{eff} \end{pmatrix} \begin{bmatrix} \frac{p'}{\rho c} \\ u' \end{bmatrix}_u$$

Eq. 6

where α is the ratio between the upstream and downstream areas, l_{eff} takes into account the inertia of the air mass in the duct and the variation of section between plenum and combustor, while the coefficient ζ gives the acoustical pressure losses and is generally close to the mean flow pressure loss coefficient. For details on the derivation or on the significance of the terms, please refer to the provided references.

4. Use of COMSOL Multiphysics

In this section the combustor, the numerical domain and the main strategies adopted in the solution of the problem are presented. Results will be discussed in the last subsection.

4.1 Geometry and computational domain

In this work, the simplified tubular combustor, studied by Dowling and Stow [4], is investigated. It is composed by three cylindrical ducts: plenum, premixer and combustion chamber. Details on the geometry and thermodynamic data are provided by Table 1 while a scheme of the combustor is in Figure 1.

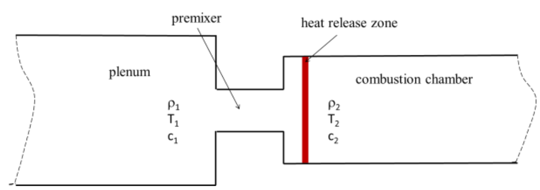

Figure 1 Simplified scheme of the investigated combustor.

Table 1 Geometry and thermo-dynamical conditions for the studied combustor

Description	***Value***
Choked inlet, temperature	300 K
Plenum, cross-sectional area	0.0129 m^2
Plenum, length	1.7 m
Premixer, cross-sectional area	0.00142 m^2
Fuel convection time delay	0.006s
Premixer length	0.0345 m
Combustor, cross-sectional area	0.00385 m^2
Flame zone, temperature after combustion	2000 K
Combustor, length	1 m
Open outlet, pressure	101 kPa

In the example it is assumed that, at the inlet, the flow is nearly choked while at the outlet an open end is considered. The thin flame zone is placed just downstream the premixer exit, at the inlet of the combustion chamber. For the fluctuating heat release, the model presented before (Eq. 4) is used, where the injection point (where to compute $\bar{u}_i$ and ρ_i) is assumed to be at the premixer inlet. The time averaged heat release for unit of area (infinitely thin flame) of the combustion chamber is

$$\bar{q} = \rho_i \bar{u}_i c_p (\bar{T}_2 - \bar{T}_1) \frac{A_{premix}}{A_{chamb}} \qquad \text{Eq. 7}$$

where c_p is the specific heat at constant pressure and $\bar{T}_1$ and $\bar{T}_2$ are the temperature in the premixer and combustion chamber respectively.
The geometry is built directly in COMSOL. The computational mesh in Figure 2 has been built choosing a maximum element size in order to detect acoustic modes up to 2 kHz.
The problem is solved in the COMSOL *Pressure Acoustics* module where, as mentioned, the inhomogeneous Helmholtz equation in Eq. 3, in the frequency domain, is solved. That is, no mean flow is considered.
In their analysis, Dowling and Stow, considered a mass flow of 0.05 kgs^{-1}. The local effects of the mean flow, in this work, are accounted for in the BTM which replaces the premixer, excluded from the simulation. The BTM is computed using the *Aeroacoustic* module in COMSOL considering three different values for the mass flow, in order to verify the effects of the Mach number. Following the procedure successfully adopted in [9], the TM is implemented in *Global Equations* and a *Normal Acceleration* boundary conditions are applied at the matrix interfaces in Figure 2 considering the acoustic velocities.

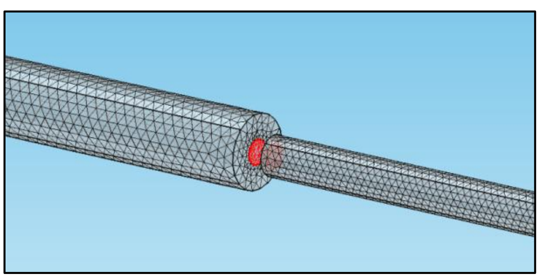

Figure 2 The computational mesh and the interfaces for the burner transfer matrix application

4.2 Burner Transfer Matrix Computation

The implemented procedure to compute the TM is the numerical implementation of the experimental two-source technique. Basically such experiments involve perturbing the element with an acoustic source as an in-line siren or wall-mounted loudspeakers. Being four the coefficients to be determined, the element needs to be perturbed from both sides, once upstream and once downstream. This approach does not rely on any modelling, assuming only linear perturbations. The main drawback is the necessity of repeating the determination procedure for each operating condition, being the TM dependent on the mean thermodynamics conditions. If no mean flow is considered the TM can be computed using the *Pressure Acoustics* module. In the present work, three different flow rates are considered: the same mass flow adopted in [4] of 0.05 kgs^{-1} as well as 0.075 and 0.1 kgs^{-1}.
A laminar flow physics is introduced in the model and solved, assigning at the inlet the desired mass flow. In Figure 3 it is possible to observe the flow-field for the 0.05 kgs^{-1} case.

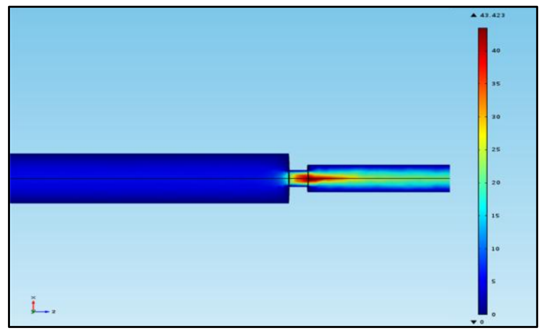

Figure 3 Mean velocity field used for BTM computation for the 0.05 kgs^-1 case

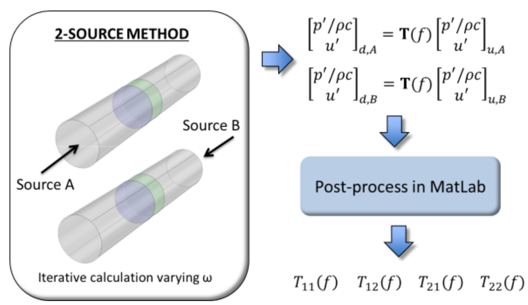

Figure 4 Scheme of the BTM computation procedure

Once the mean flow is solved, a frequency domain simulation with acoustic forcing is performed, varying the frequency in the desired range and recording the values for pressure and velocity fluctuations at the TM interfaces. A dedicated Matlab™ post-process procedure is then used to retrieve the matrix coefficients (see Figure 4).

4.3 Results

The system is simulated at first without heat release, that is, setting k=0 in Eq. 4. In Figure 5 are reported the results obtained adopting the

BTM, computed with the three tested flow rates (red). The blue symbols are, instead, the results obtained with the reference in-house 1D code (1D-C). Black dots are the case without TM while green squares the analytical solution by [4] (Ref.).

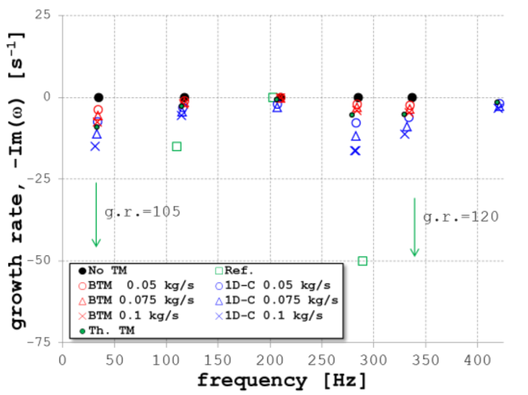

Figure 5 Resonant modes of the combustor for no unsteady heat release (k = 0)

The resonant modes found for the plenum are predicted at 116 Hz, 210 Hz, and 284 Hz while the first mode of the combustion chamber is at 335 Hz. The lowest mode at 34 Hz is the first of the entire combustor. The BTM approach is able to correctly predict the dissipative effect through a decreasing growth rate (g.r.) with an increasing mean flow. In all the cases a negative g.r. is found, meaning stable modes. Predicted g.r. are higher than the same computed with the 1D code. In any case, the values provided by [4] are not matched in most cases.

On the same plot in Figure 5, are reported the results obtained with the theoretic TM (Th. TM) in Eq. 6. It emerges that, acting on the parameters ξ and l_{eff} (1 and 0.0345 m in this work), it is possible to find a setting able to reproduce the obtained trends for the modes. A consideration must be done: the use of such a TM might reveal very useful in a design phase, where the effects of the different parameters (e.g. ξ) is to be investigated. In case of analysis there might be too many degree of freedom. In fact, without a reference point for the calibration of the matrix coefficient, it is not straightforward finding a reliable setup. In this last case, a computed transfer matrix approach might be more suitable, as it does not introduce any modelling hypothesis, against the necessity of computing a new TM for each operating condition.

The unsteady heat release is then introduced, setting k=1.

Simulations are first performed maintaining a constant τ= 0.006s, in order to appreciate the effect of the only mean flow, in the presence of unsteady heat release. The principal modes are well represented in COMSOL (see Figure 7), also for that family of modes related to the flame model, i.e. 166-Hz mode. As pointed out in [4], these modes are strictly connected with the τ and their frequency is around 1/ τ, 2/ τ etc.

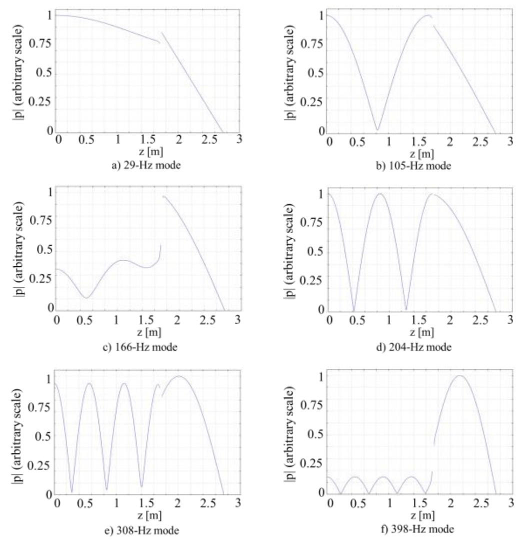

Figure 6 Mode shapes of the computed modes for the case BTM 0.05kg s-1 and τ = 0.006s

The shape predicted by COMSOL model, for some of the computed modes, with BTM, 0.05kg s-1 and τ = 0.006s, are reported in Figure 6 and Figure 7. It is clear that the mode b), d), e) belongs to the family of the plenum resonant modes. Mode c) and f) are the additional modes which are found in case of non-zero τ. The first mode of the combustor is the one predicted around 298 Hz.

In Figure 8 the shape of the first mode obtained for the cases Th. TM, BTM 0.05kgs^{-1} and BTM 0.1 kgs^{-1} are compared with that computed analytically in [4]. It is possible to see how, increasing the flow rate from c) to d) the blockage effect at the premixer exit is increased. The effect of mean flow is to decouple the connected part of the plenum and combustion

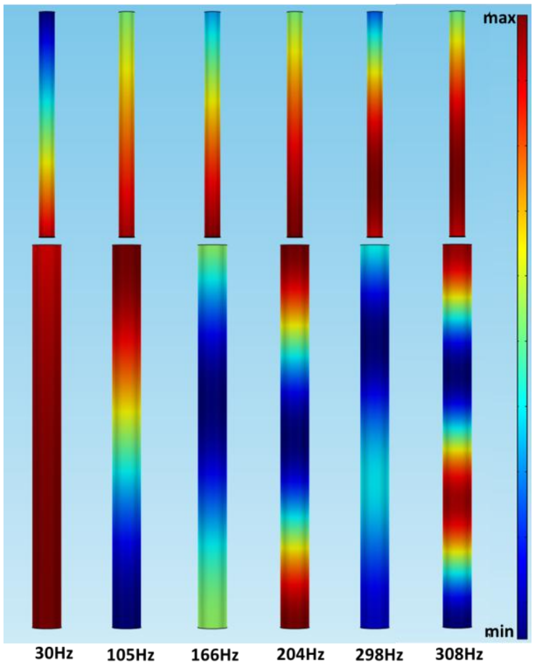

Figure 7 Obtained modes for the BTM 0.05kgs-1 case

chamber. If compared with the analytical solution (a), case c) predicts a lower decoupling effect. This is likely to be due to the fact that, computing the BTM with the laminar solver, the effects of turbulence on the dissipation are neglected, thus underestimating the impact of the mass flow.

The theoretical TM b) predicts a higher blockage if compared with the case at the same mean flow c). Calibrating the TM coefficient the mode shape might have been better reproduced but altering (loering) the growth rate in the process.

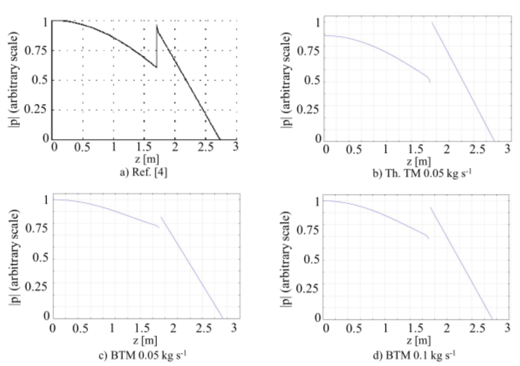

Figure 8 First mode shape for three of the computed cases and analytical solution **[4].**

The stabilizing effect of the mean flow is properly predicted (Figure 9) and results are in line with the 1D code ones. As far as the first frequency is concerned, a discrepancy emerges for the first mode between the solvers. In particular, when the flame effect is introduced, a different trend for the value of the frequency is predicted. While the FEM solver predicts a growing frequency with the flow rate, in the 1D code a slight decrease is found. A possible explanation is in terms of entropy wave, not solved in COMSOL, which, as stated, might have an impact on the lower frequencies of the system. When no entropy is introduced in the system by the flame (k=0 case in Figure 5), in fact, both the simulations predict the same trend.

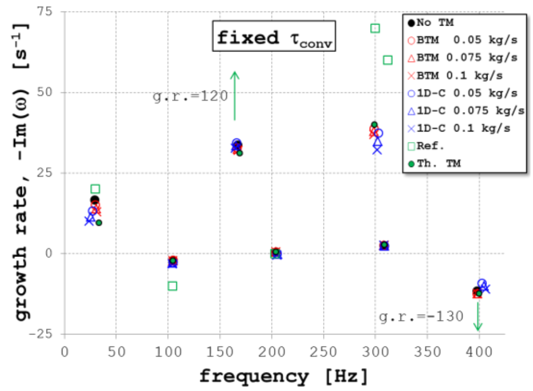

Figure 9 Resonant modes for the combustor obtained maintaining a fixed time delay ($\tau = 0.006$s)

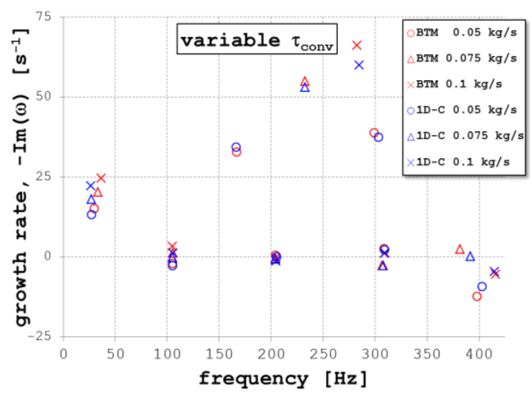

Figure 10 Resonant modes for the combustor obtained varying the time delay with the mean flow

In the last set of simulations the delay time, computed as a convective time from the injection point to the flame region, is varied according to the mean flow in the simulation. Again, a good agreement with the reference simulation is achieved. The modes and growth rates are well predicted. In contrast to the simulation at constant τ, where the effect of an increased mean flow was a stabilization of all the frequencies, now some of the mode is destabilized (i.e ~34 Hz mode). In other words, the effect of mean flow on the convective delay time has a greater influence than the enhanced dissipation. BTM computed modes are found to be more unstable than the 1D code predicted one. This is likely due to some stabilizing effect, as the losses due to an area change, not introduced in the FEM simulation by the BTM approach.

5. Conclusions

A methodology to include mean flow effects in the acoustic computation of a tubular combustor, using a burner transfer matrix approach, has been successfully exploited. The procedure has been assessed and tested. The effects of a mean flow have been evaluated in terms of dissipation and blockage at the premixer section as well as in terms of growth rate and mode shape of the computed eigenfrequencies. Comparisons with a 1D in-house code, theoretical BTM formulation and analytical solutions available in literature showed a good prediction of the trends but an underestimation of the dissipative effect. Improvement in the BTM computation should be led by the introduction of turbulence effects, neglected in the actual BTM computation.

6. References

1. A. Andreini, B. Facchini, A. Giusti, F. Turrini and I. Vitale, Thermo-Acoustic Analysis of an Advanced Lean Injection System in a Tubular Combustor Configuration, *Proceedings of the COMSOL Conference 2012 Milan*, (2012).
2. F. Nicoud and K. Wieczorek, About the zero mach number assumption in the calculation of thermoacoustic instabilities, **vol. 1**, no. 1, (2009).
3. B. T. Chu, On the energy transfer to small disturbances in fluid flow (Part I), *Acta Mechanica*, pp. 215-234, (1965).
4. A. P. Dowling and S. R. Stow, Acoustic analysis of gas turbine combustors, *Journal of propulsion and power*, **vol. 5**, no. 19, pp. 751-765, (2003).
5. L. Kovasznay, Turbulence in supersonic flow, J. Aeronaut. Sci., **vol. 20**, pp. 657-674, (1953).
6. E. Motheau, L. Selle and F. Nicoud, Accounting for convective effects in zero-Mach number thermoacoustic models, *Journal of sound and vibration*, **vol. 333**, no. 1, pp. 246-262, (2014).
7. D. Fanaca, P. Alemela, F. Ettner, C. Hirsch, T. Sattelmayer and B. Schuermans, Determination and comparison of the dynamic characteristics of a perfectly premixed flame in both single and annular combustion chamber, *ASME paper*, **GT2008-50781**, (2008).
8. P. Alemela, D. Fanaca, F. Ettner, C. Hirsch, T. Sattelmayer and B. Schuermans, Flame transfer matrices of a premixed flame and a global check with modeling and experiments, *ASME paper*, **GT2008-50111**, (2008).
9. G. Campa and S. M. Camporeale, Application of Transfer Matrix Method in Acoustics, *Proceedings of the COMSOL Conference 2010 Paris*, (2010).
10. F. E. Marble and S. Candel, Acoustic disturbances from gas nonuniformities convected through a nozzle, *J. Sound Vib.*, **vol. 55**, pp. 225-243, (1977).