# Validation of measurement strategies and anisotropic models used in electrical reconstructions

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**Abstract:** We are developing approximations of electrically anisotropic materials for use in novel imaging methods. Our modeling work comprises comparisons of anisotropic and layered models in terms of electrical conductivities measured using different strategies. We tested solution stability in one anisotropic case by varying mesh anisotropy. We found that in our case, good approximations to the true anisotropic solutions were usually found only at extremely fine mesh levels.

**Keywords:** Conductivity Tensor, Anisotropic mesh, EIT, MREIT.

## 1. Introduction

We are interested in electrically modeling the human skull and need to determine if the skull is best considered as a layered isotropic or homogeneous anisotropic electrical component. We are also interested in using finely layered gels as approximately anisotropic test materials for electrical and magnetic resonance imaging methods.

We characterize the anisotropic conductivity tensor as having components tangential and radial to the slab face (denoted $\sigma_t$, and $\sigma_r$). In three dimensions, the tensor will be

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_t & 0 & 0 \\ 0 & \sigma_r & 0 \\ 0 & 0 & \sigma_t \end{bmatrix} \quad (1)$$

Where the material is aligned with the axes as shown in Figure 1(a).

Anisotropic materials can be approximated by layering alternate high and low conductivity slabs. The highest conductivity direction will be tangential to the slab faces. As more, thinner, slices are added while keeping the overall height of the construction constant, the approximation to anisotropic behavior becomes better. Figure 1(b) shows an electrode arrangement that can be used on a layered or anisotropic sample to characterize its electrical behavior. With four electrodes, two used for applying current and two for measuring voltage, there are three different conductivities that can be measured. In an inhomogeneous or anisotropic sample the conductivity measured in each case will be related to both a cell constant relative to the sample and electrode geometry, and also to a factor that depends on the degree of anisotropy in the material.

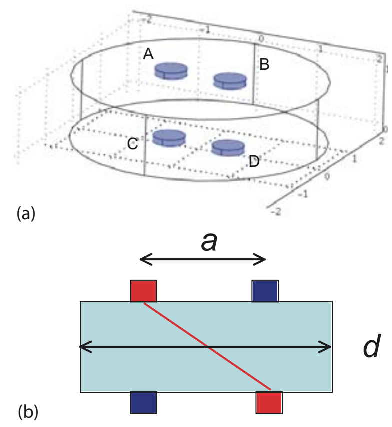


**Figure 1** (a) Three-dimensional positions of electrodes on sample (b) diagonal measurement strategy, with voltage or current application electrodes shown in red, and measurement electrodes shown in blue. The distance between electrodes is *a*. The height of the sample (in the y direction) is *h*. The case studied in this paper was *a* = *h*.

We choose to concentrate on the measurement shown in Figure 1 (b), which involves passing current obliquely between electrodes on top and bottom faces of the sample. We term this the diagonal measurement. These measurements are particularly sensitive to anisotropy in the sample. An illustration of the variation in diagonal measurement field patterns occurring within the sample as anisotropy is increased is shown in Figure 2. Note that an anisotropy ratio ($k = \sigma_t/\sigma_r$) of approximately 2, a voltage isosurface passes through the two measurement electrodes. There is no potential difference between the electrodes and therefore the resistance calculated by the formula

$$R_{app} = \frac{V_{m2} - V_{m1}}{I} \quad (2)$$

where $R_{app}$ is the apparent resistance, $V_{m1}$ and $V_{m2}$ are the voltages at measurement electrodes 1 and 2 respectively and $I$ is the current applied through the other two electrodes, will be zero. Thus, at this degree of anisotropy the apparent resistivity of the sample is zero and the apparent conductivity is infinite. As $k$ increases further the polarity of voltages at the two measurement electrodes is swapped and voltages again become non-zero, as shown by the change in shape of the zero-volt isosurface in Figure 2.

Diagonal measurements are highly sensitive to material composition. We chose to study the properties of a finite element model of an anisotropic slab as anisotropy in the $x$ and $z$ directions was increased.

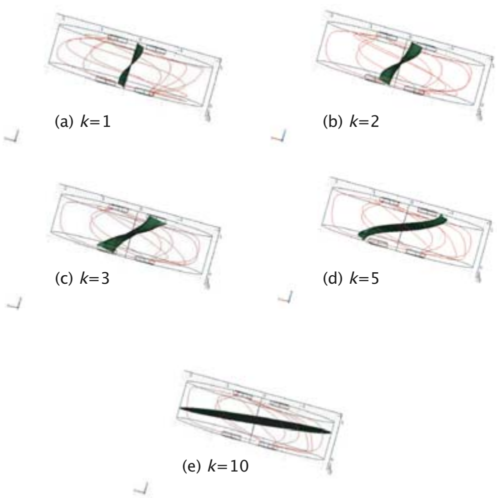


**Figure 2** Current streamlines (red) and zero-volt isosurfaces (green) plotted for anisotropy ratios k of (a) 1 (isotropy), (b) 2, (c) 3, (d) 5 and (e) 10, showing a minimum I differential voltage at around $k = 2$.

It is difficult to find an appropriate analytic solution for the voltage formed between the two measurement electrodes. It is possible to formulate an expression in the case of an infinite slab with finite thickness (Sadleir 2007, Rush 1962, Livshitz 2000) but comparisons to real models are difficult because of dependencies on object size and electrode size. A comparison of diagonal analytic and accurate finite element apparent conductivities as anisotropy is varied is shown in Figure 3 below.

Use of anisotropic meshes is recommended when meshing thin structures or in cases where the solution metric is greatly different from the object geometry. We were curious to determine if accurate and efficient solutions could be obtained by varying mesh anisotropy. We believed that we might be able to determine that a solution was accurate when results obtained using different degrees of mesh anisotropy were similar.

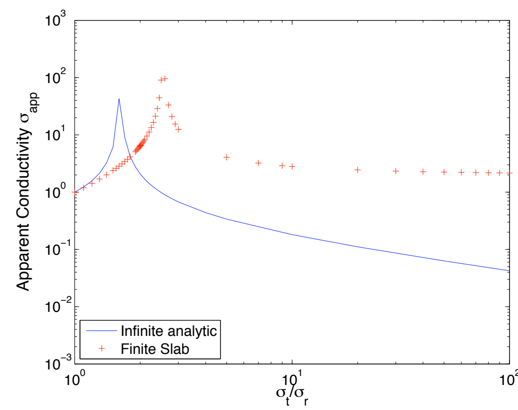


**Figure 3** Apparent conductivities calculated using diagonal configuration with varying degrees of anisotropy using (blue) an analytic model of an infinite finite thickness slab and (red) a finite anisotropic block with finite-sized electrodes.

## 2. Methods

We constructed an anisotropic block with dimensions *4.4 m* x *1.2 m* x *3 m* using COMSOL 3.4. We placed four electrodes on the sample, two on the top and two on the bottom, with the two electrodes on the same face separated along the $x$ direction by 1.2 m (the same distance as the $y$ dimension of the block). Using the diagonal measurement configuration, we applied constant voltages of ± 1 V to one pair and determined resulting voltages on the other electrode pair. Current flowing through the two electrodes at fixed voltages was determined and the resistance measured using (2). After solving the model using a geometric multigrid (GMG) at a range of different values for $\sigma_t$, we remeshed the model using the different mesh fineness settings defined by COMSOL and resolved models. We then meshed at three different fineness levels with a mesh anisotropy in the *y* direction that was 20 times that in the *x* and *z* directions. The different mesh levels considered are shown in Table 1.

**Table 1** Models considered for comparison of diagonal solutions, showing number of elements, degrees of freedom (DOF), time for solution and number of geometric multigrid iterations required.

| Model Name | Elements | DOF | Time (s) | GMG iterations |
|---|---|---|---|---|
| ecoarse | 759 | 1374 | 0.21 | 20 |
| coarser | 1282 | 2239 | 0.27 | 18 |
| normal | 5312 | 8549 | 0.74 | 17 |
| finer | 21960 | 32796 | 3 | 19 |
| efine | 413271 | 574266 | 83 | 18 |
| normal20y | 4815 | 7711 | 3 | 271 |
| finer20y | 20490 | 33130 | 19 | 347 |
| efine20y | 142668 | 214912 | 120 | 287 |

## 3. Results

We found that for the model tested, the normal, finer and efine levels of mesh fineness gave similar results for the anisotropic measures. However, when mesh anisotropy was included in the normal and finer models (while keeping the total number of elements approximately constant) the zero resistance point was significantly shifted. This was not observed for the efine20y case, even though significantly fewer elements were generated.

All anisotropic mesh cases took a great deal longer than the isotropic mesh cases to solve.

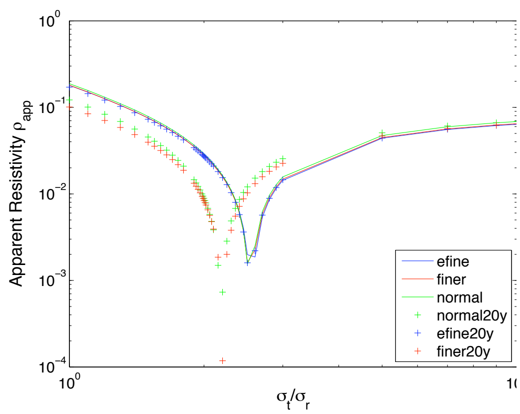


**Figure 4**. Comparison of Apparent Resistivities observed using the diagonal measurement configuration and the three finest models, compared with results obtained using their corresponding anisotropic meshes, as a function of material anisotropy *k*.

Results obtained using the ecoarse and coarser models were very different from the three fine isotropic mesh models, as shown in Figure 5.

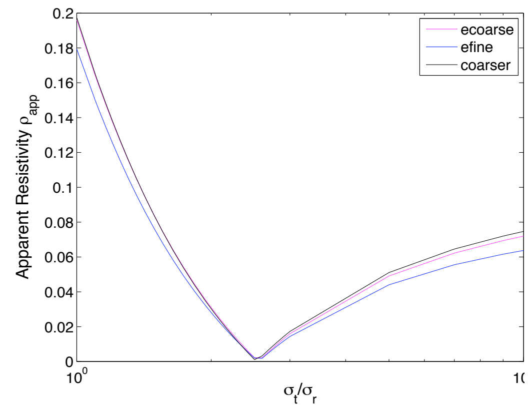


**Figure 5.** Results for diagonal measurements on Ecoarse and Coarse models, as a function of material anisotropy, compared with Efine results

## 4. Discussion

The amount of variability between ecoarse and coarser models compared with the efine model probably indicates that these solutions are not very accurate. We observed that the metric effectively changed when anisotropic material and anisotropic meshes were combined at low levels of mesh fineness (normal and finer mesh levels). However, at the efine level good agreement was observed between results generated with both and isotropic and anisotropic meshes. This method may be a good one for testing solution robustness for complex model geometries and material. It is interesting that solutions for normal and finer mesh levels also agree well with efine solutions when isotropic meshing is used. This is the subject of further investigation.

## 5. Conclusions

Anisotropic material properties are particularly vulnerable to errors in mesh scaling. Even if solutions look reasonable, a useful method of testing for solution accuracy in complex cases may be to vary mesh scaling. We found that for the model tested, good solutions on materials with anisotropic material properties were observed using extremely fine mesh settings and approximately 500 000 elements.

## 6. References

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## 7. Acknowledgements

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