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Polynomial-Chaos Uncertainty Modeling in Eddy-Current Inspection of Cracks

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Introduction

- ✓ We are interested in studying the problem of ***eddy-current inspection of cracks***, when ***geometric uncertainties*** are present.
- ✓ Till today, problems involving eddy currents are rarely solved computationally in a ***stochastic framework***.
- ✓ ***FEM modeling*** via the COMSOL® software is exploited, combined with ***Matlab scripting***.
- ✓ ***Monte-Carlo*** (MC) methodologies are ***computationally inefficient***, due to ***slow convergence***.

Uncertainty quantification is performed here in a ***non-intrusive*** fashion, by computing ***polynomial-chaos (PC) expansions*** of the random output quantities in an efficient manner that adopts ***sparse-grid quadrature*** schemes.

Reliable statistical information is extracted with a ***reduced number of simulations***.

Problem Description

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\mathbf{A}\right)+\left(\mathrm{j}\omega\sigma-\omega^2\varepsilon\right)\mathbf{A}=\mathbf{J}_s \ \text{ in } \Omega,\ \hat{\mathbf{n}}\times\mathbf{A}=\mathbf{0}\ \ at\ \ \partial\Omega$$

Current-source density

$$\mathbf{J}_s=J_0\frac{-y\hat{\mathbf{x}}+x\hat{\mathbf{y}}}{\sqrt{x^2+y^2}}$$

Coil-impedance

$$Z=R+\mathrm{j}X=\frac{\mathrm{j}\omega}{I^2}\iiint_{V_c}\mathbf{A}\cdot\mathbf{J}dV$$

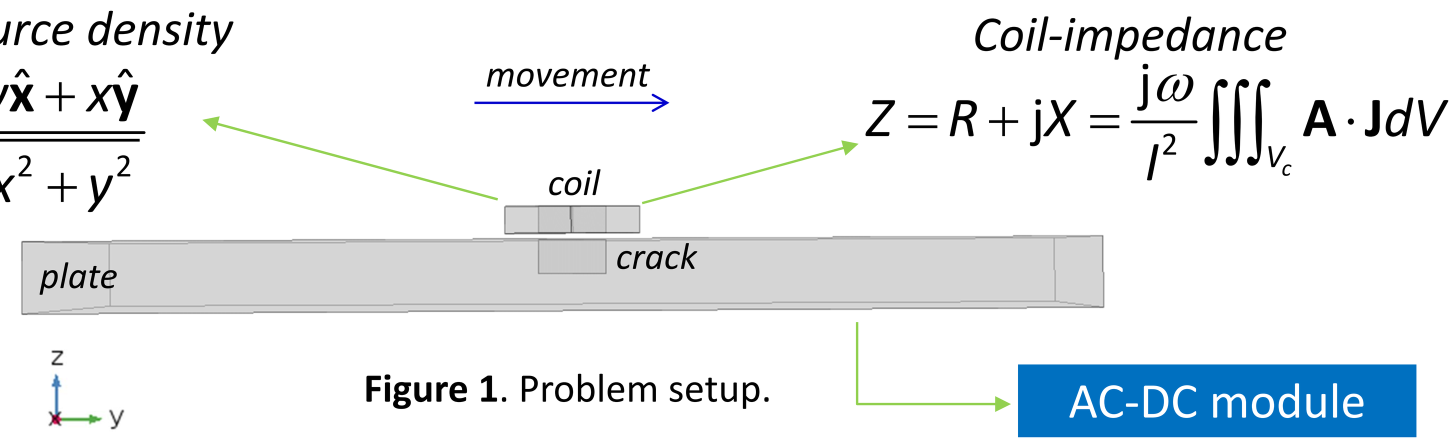


Figure 1. Problem setup.

Polynomial Chaos Models

***p*-th order PC series of ΔR, ΔX for *d* random variables**

$$P+1=\frac{(p+d)!}{p!d!}$$

$$\Delta U(\mathbf{r},\boldsymbol{\xi})\simeq\sum_{\ell=0}^{P}C_\ell^U(\mathbf{r})\Psi_\ell(\boldsymbol{\xi})$$

Expansion coefficients, depending on coil position only.

$$\Psi_\ell(\boldsymbol{\xi})=\psi_{\alpha_1^\ell}(\xi_1)\cdot\psi_{\alpha_2^\ell}(\xi_2)\cdot\ldots\cdot\psi_{\alpha_d^\ell}(\xi_d)$$

Legendre *polynomials for uniform variables*

Orthogonality property, with respect to $\langle f(\boldsymbol{\xi}),g(\boldsymbol{\xi})\rangle=\int_{\Gamma^d}f(\boldsymbol{\xi})g(\boldsymbol{\xi})p(\boldsymbol{\xi})d\boldsymbol{\xi}$

$$\left\langle\Psi_i(\boldsymbol{\xi}),\Psi_j(\boldsymbol{\xi})\right\rangle=\left\|\Psi_i(\boldsymbol{\xi})\right\|^2\delta_{ij}$$

Calculation of Expansion Coefficients

Spectral projection

$$C_k^U(\mathbf{r})=\frac{\left\langle\Delta U(\mathbf{r},\boldsymbol{\xi}),\Psi_k(\boldsymbol{\xi})\right\rangle}{\left\|\Psi_k(\boldsymbol{\xi})\right\|^2},\ k=0,\ldots,P$$

Quadrature

$$\left\langle\Delta U(\mathbf{r},\boldsymbol{\xi}),\Psi_k(\boldsymbol{\xi})\right\rangle\simeq\sum_{i=1}^{N}w\left(\boldsymbol{\xi}^{(i)}\right)\Delta U\left(\mathbf{r},\boldsymbol{\xi}^{(i)}\right)\Psi_k\left(\boldsymbol{\xi}^{(i)}\right)p\left(\boldsymbol{\xi}^{(i)}\right)$$

Sparse-grid (Smolyak) approach

grid construction — *quadrature rule*

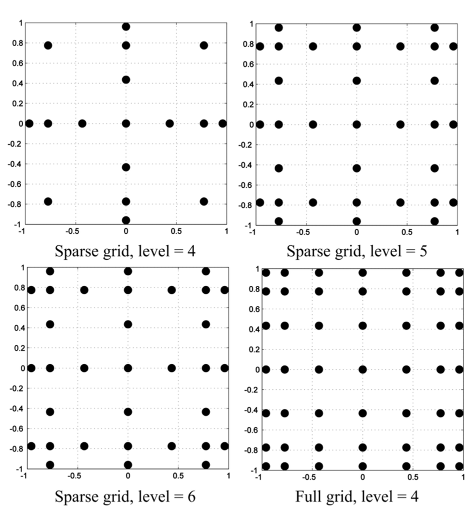
$$\Theta=\bigcup_{k+1\le|\mathbf{i}|\le k+d}\left(\Theta_1^{i_1}\times\Theta_1^{i_2}\times\ldots\times\Theta_1^{i_d}\right)\qquad I\simeq\sum_{k+1\le|\mathbf{i}|\le k+d}(-1)^{k+d-|\mathbf{i}|}\binom{d-1}{k+d-|\mathbf{i}|}\left(U^{i_1}\otimes U^{i_2}\otimes\ldots\otimes U^{i_d}\right)$$

Delayed Kronrod-Patterson nodal sets — *selected tensor-product grids*

Mean value $\quad E\left[\Delta U\right]=C_0^U$

Variance $\quad \mathrm{var}\left[\Delta U\right]=\sum_{k=1}^{P}\left(C_k^U\right)^2\left\|\Psi_k(\boldsymbol{\xi})\right\|^2$

Direct calculation of standard statistical norms from the expansion coefficients



Matlab directives

```
function out = coil_defect(m,xc,yc,zc)
...
model.param.set('m',[num2str(m),
'[mm]'], 'movement');
model.param.set('xc',[num2str(xc),
'[mm]'], 'x dimension of crack');
model.param.set('yc',[num2str(yc),
'[mm]'], 'y dimension of crack');
model.param.set('zc',[num2str(zc),
'[mm]'], 'z dimension of crack');
```

Computational cost

2000 MC samples require approximately **6.3 days** on an i7-4820K CPU @ 3.7 GHz!

Numerical Results

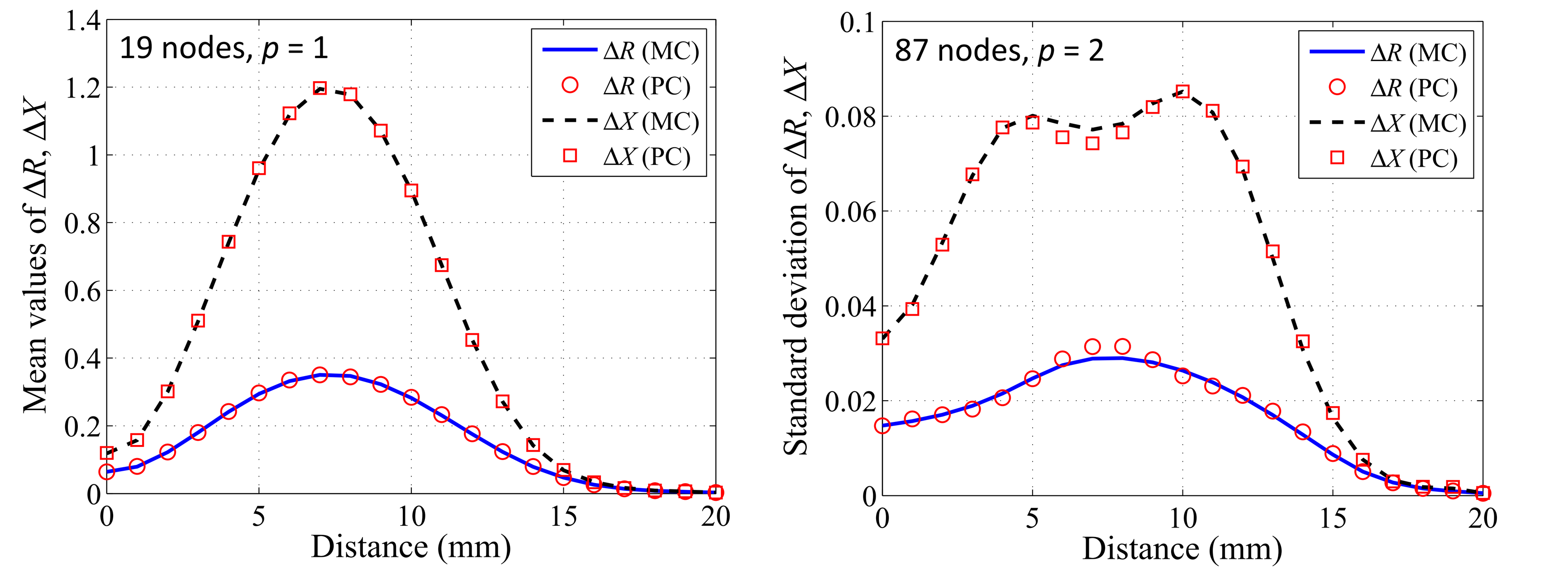


Figure 2. Mean value (left) and standard deviation (right) of the change in coil's impedance, as a function of the coil position.

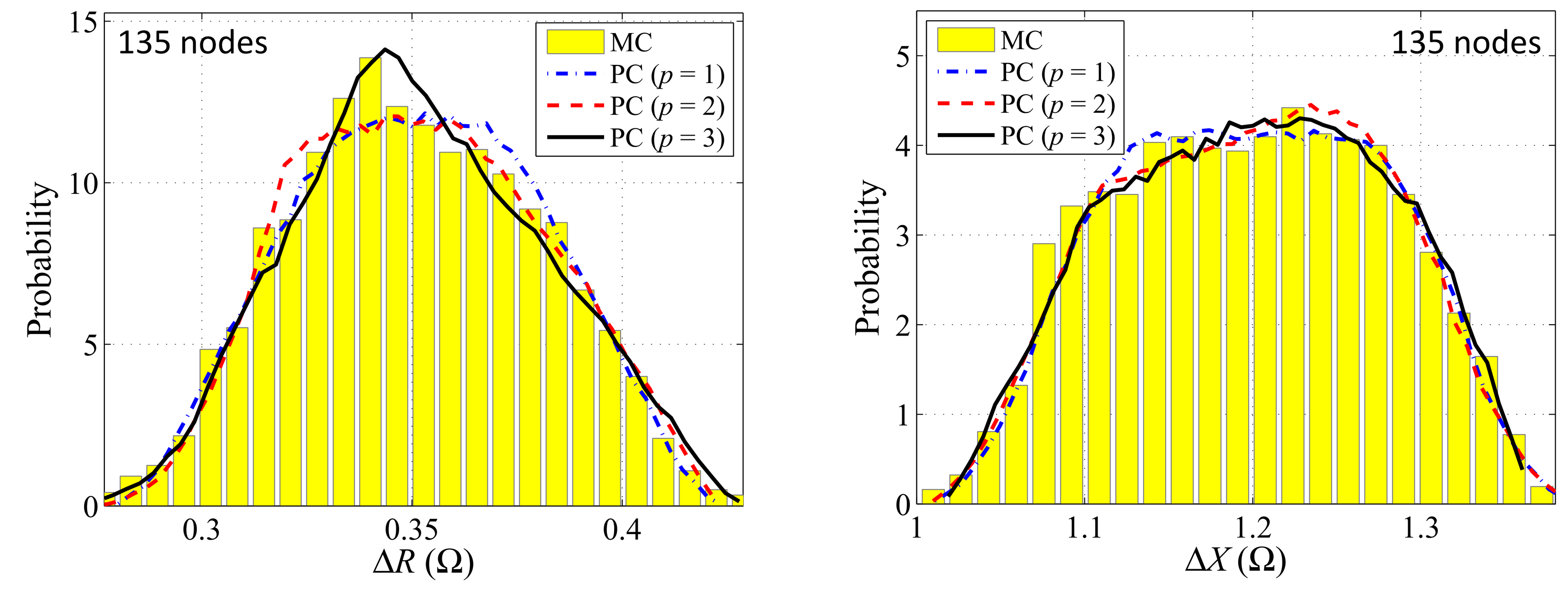


Figure 3. Computed PDFs corresponding to ΔR (left) and ΔX (right) at a distance of 7 mm.

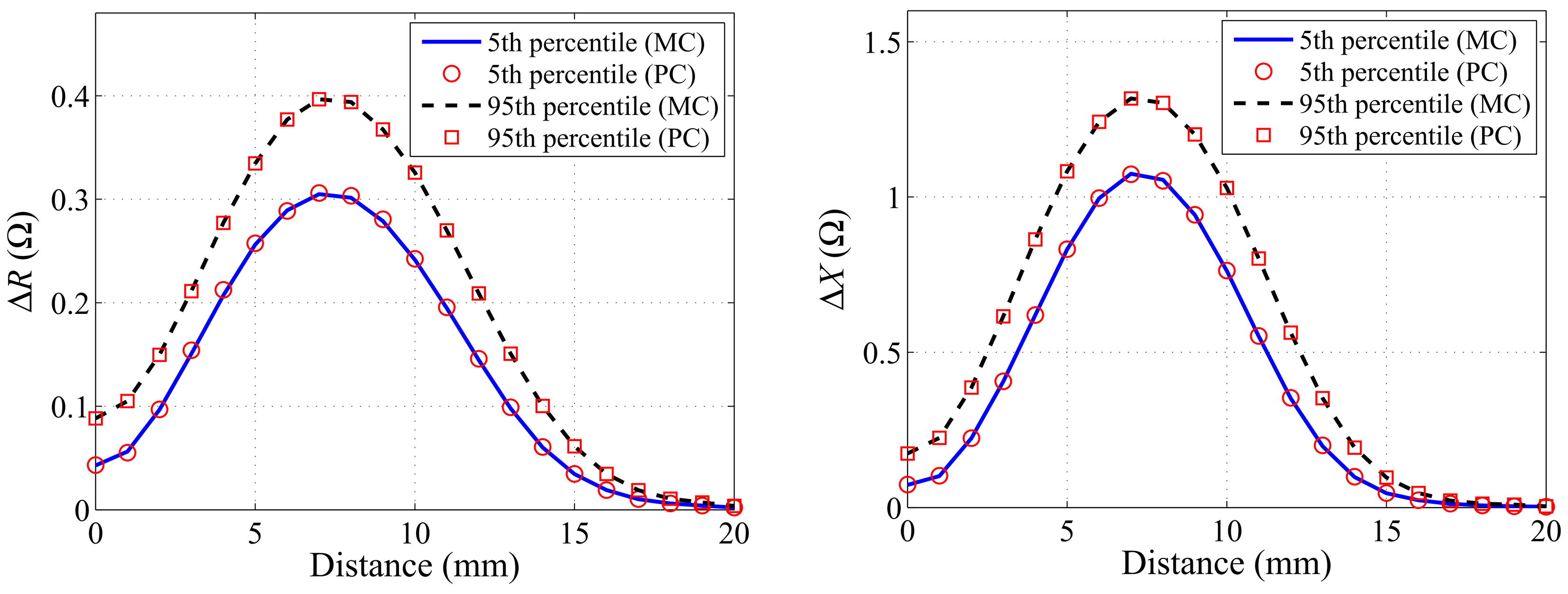


Figure 4. Upper and lower bounds of ΔR and ΔX, represented by the 95th and 5th percentiles.

Sensitivity Analysis

- Direct calculation of ***Sobol indices*** from the PC coefficients.

$$S_v^U\simeq\frac{\sum_{\ell\in K_v}\left(C_\ell^U\right)^2\left\|\Psi_\ell\right\|^2}{\sum_{\ell=1}^{P}\left(C_\ell^U\right)^2\left\|\Psi_\ell\right\|^2}$$

$v\in\{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\}$

The significant influence of the ***crack's length*** on the output variability is revealed

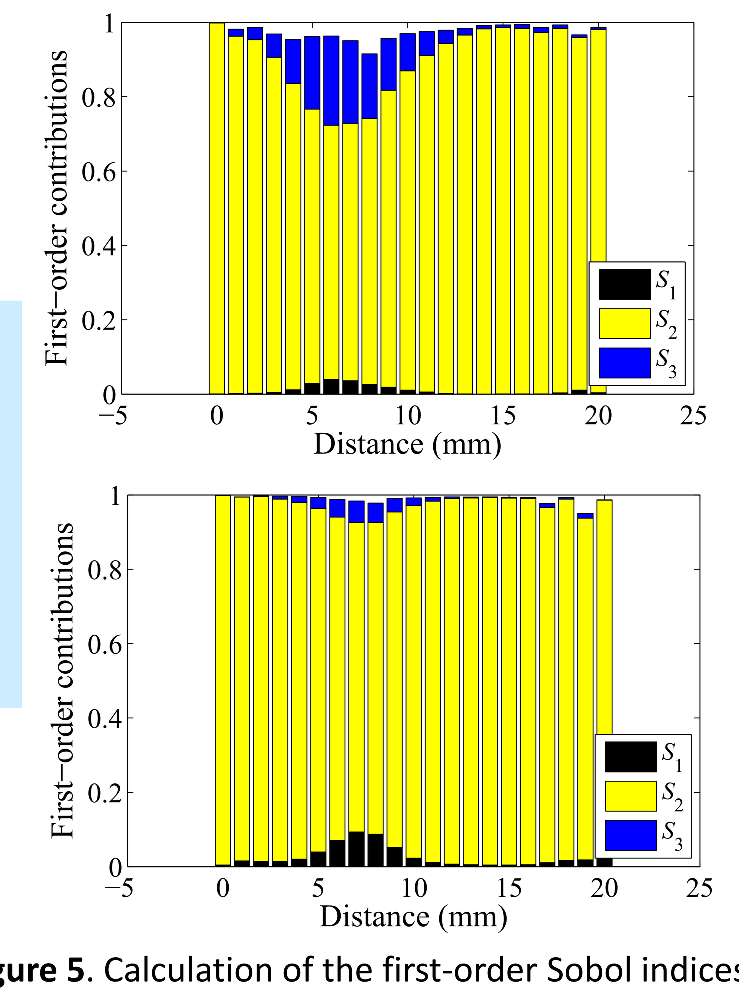


Figure 5. Calculation of the first-order Sobol indices, regarding the real part (top) and the imaginary part (bottom) of the coil's impedance change.

Conclusion

- ✓ The ***deterministic*** FEM solver of COMSOL® has been used for the investigation of ***stochastic eddy-current testing*** problems.
- ✓ The PC approach provides ***reliable*** statistical information using a ***fraction*** of MC's computational requirements.
- ✓ Uncertainty in the ***length of the defect*** appears to have the most ***significant impact*** on the impedance variability.
- ✓ A ±10% uncertainty in the three geometric parameters may induce a deviation in the range 8-22% in ΔR and in 5-25% in ΔX.

References

1. L. Santandrea and Y. Le Bihan, Using COMSOL-multiphysics in an eddy current non-destructive testing context, *Proceedings of the COMSOL Conference*, Paris (2010).
2. D. Xiu and G. E. Karniadakis, The Wiener–Askey polynomial chaos for stochastic differential equations, *SIAM J. Sci. Comp.*, 24 (2), 619-644 (2002).
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