Excerpt from the Proceedings of the COMSOL Conference 2009 Milan

A Semplified Model for the Evolution of a Geothermal Field

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Abstract: The problem is to understand how a geothermal field can evolve from a water dominated state (i.e. when the geothermal fluid is mainly in liquid phase) into a vapour dominated one. A first answer to this question is given by a semplified mathematical model of the dynamics of a geothermal field in which the geothermal fluid is entirely composed by pure H_2O. We considered a one-dimensional geometry and we developed a dynamic model that presents a clear interface between the gas phase (which occupies the upper part of the basin) and the liquid phase (which occupies the lower part). Due to the process of evaporation or condensation, the interface changes its position over time and it is therefore modeled with a free boundary, whose dynamics is governed by Rankine-Hugoniot conditions. We obtained a **free boundary problem** of Stefan type. Solving the problem by COMSOL Multiphysics, it is possible to get informations on the evolution of the interface.

Keywords: Geothermal field, free boundary problem, moving mesh, ALE method.

1 Introduction

A geothermal reservoir is a complex permeable medium, located deep in the earth's crust, consisting in fracturated hot rocks in which the geothermal fluid, composed primarily of water, can flow. The latter is responsible of the heat transfer and it can achieve the ground surface, due to natural way (geyser, "fumarole", etc.) or due to industrial purposes (geothermal wells). According to the predominant phase of the geothermal fluid the geothermal reservoirs can be divided into *water dominated* or *vapor dominated*. In general, a water dominated geothermal field evolves into a vapour dominated one. Examples of this evolutionary path are encountered in nature, as in the geothermal field of Larderello in Italy. Understanding this evolution is important because it allows to have information on the age of the basin and its ability to be exploited for the production of geothermal energy.

Thus, the following questions naturally arise:

a) Which are the main factors affecting the evolution of a geothermal field?

b) What is the geological time evolution of a water-dominated basin into a vapor dominated one?

In an attempt to shed light on these questions, we shall discuss in this paper a simplified mathematical model, which however displays the physical phenomenon explained above.

This work is organized as follows: Section 2 is devoted to describe the physical assumptions, the schematization of the problem and the mathematical modeling of the phenomenon. In Section 3 we show the numerical techniques exploited to solve the problem and the use of COMSOL Multiphysics. Finally, in Section 4 we give the outcome of the simulations and we discuss the results that allow us to answer the questions that we posed.

2 Mathematical Model and Governing Equations

Let us consider the geothermal fluid contain only pure H_2O in the states gaseous and liquid. Consider a one-dimensional geometry as in Figure 1. The vertical axis x cross the permeable area of the basin. At depth $x = Li$ ($Li < 0$) we are at the base of the reservoir while at depth $x = Ls$ ($Ls < 0$) at the top. The surface soil corresponds to

$x = 0$. The domain of interest is therefore the interval $[Li, Ls]$. The low part domain is saturated by liquid H_2O, unlike the top where there is only vapor. Suppose that there is a clear separation between the two zones, through a boundary $s(t)$ which may change over time. Moreover, we assume the following:

- The porosity ϕ is constant.
- The permeability K of the porous medium is independent of temperature.
- The temperature T is assigned as a linear function increasing with depth.

Besides, at the upper edge of the reservoir ($x = Ls$) let us consider a given pressure and temperature, while at the bottom ($x = Li$) a no-flow condition for liquid water is assumed.

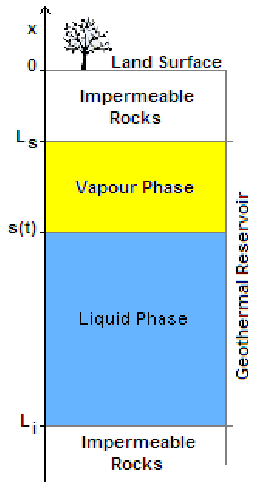

Figure 1: Sketch of the problem.

2.1 Equations

Considering the equation of continuity of mass and Darcy's law for porous media, because the hypothesis of no flux on the bottom we get the following problem for the pressure P_l of liquid water

$$\begin{cases} \dfrac{\partial P_l}{\partial x} = -\rho_l g, \\[2ex] P_l(x = s(t)) = P_l(s(t)), \end{cases} \tag{1}$$

in the domain $L_i < x < s(t)$, where ρ_l is the density of liquid H_2O e g is the acceleration of gravity. The linear solution of (1) describes the field of liquid fluid pressure P_l as follows

$$P_l(x) = P_l(s(t)) + \rho_l g(s(t) - x). \tag{2}$$

The problem then reduces to describe the pressure field in the area of steam. But, in this case, we can not find an explicit solution. The model consists of the following system of equations for the vapour pressure P_v in the variable domain $(s(t), L_s)$

$$\begin{cases} \dfrac{\partial P_v}{\partial t} - \dfrac{K\,T}{\phi\mu_v}\dfrac{\partial}{\partial x}\left[\dfrac{P_v}{T}\left(\dfrac{\partial P_v}{\partial x} + \dfrac{g}{r}\dfrac{P_v}{T}\right)\right] = 0, \\[2ex] P_v(x = L_s) = P_s, \\[2ex] P_v(x = s(t)) = P^*(s(t)), \\[2ex] \dot{s} = \dfrac{P^*(s(t))}{rT\rho_l}\dfrac{K}{\phi\mu_v}\left(\dfrac{\partial P_v}{\partial x} + \dfrac{P^*(s(t))}{rT}g\right)\Bigg|_{x=s(t)}, \\[2ex] P_v(t = 0) = P_{in}(x), \\[2ex] s(t = 0) = s_{in}, \end{cases} \tag{3}$$

where r and P^* are the constant of perfect gases and the pressure of saturated vapour (Clapeyron's pressure) respectively. Moreover, we suppose the viscosity of vapour μ_v to be constant. We notice that $(3)_1$ comes from the balance of the mass assuming Darcy's law for porous media [1], $(3)_2$ e $(3)_3$ are boundary conditions of Dirichlet type and $(1)_5$ e $(1)_6$ are the initial conditions. Equation $(3)_4$ governs the interface evolution and it arises from continuity of mass flow and momentum through the interface, the Rankine-Hugoniot conditions [2]. Hence, the model (3) obtained is a free boundary problem of Stefan type. This problem can be solved analytically in special configurations e.g., when the characteristic time of the interface is several orders of magnitude greater than the characteristic times of the widespread phenomenon of the geothermal fluid. In such picture a quasi-stationary approximation can be applied (see [3] for more details). Otherwise, problem (3) can be solved only by numerical methods.

3 Numerical Method and Use of COMSOL Multiphysics

The mathematical model (3) consists of nonlinear parabolic PDEs coupled with the free boundary equation and endowed with suitable boundary conditions. We opted for a numerical technique based on the Finite Element Method. To simulate the movement of the spatial domain, we exploited the technique of *deformed mesh*. The key point is that, instead of generating a new mesh for each configuration of the boundaries, we use a technique that perturbs the mesh nodes conforming them with the moved boundaries. In COMSOL Multiphysics this method is implemented in the package **ALE for Moving Mesh**. The ALE method is an intermediate between the Lagrangian and the Eulerian method, and it combines the best features of both: it allows moving boundaries without the need for the mesh movement to follow the material [4]. In our case, we set the free boundary equation $(3)_4$ into the "Mesh velocity" field of the Boundary Setting window of ALE package (see Figure 2). The successful implementation accounts for COMSOL potential to solve free boundary problems.

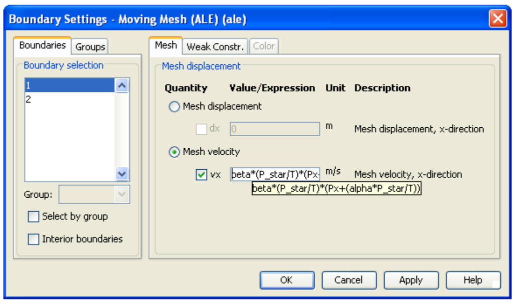

Figure 2: Boundary Setting window of tool Moving Mesh where setting the free boundary condition.

4 Results and Conclusions

The model allows to describe the variation of field of vapour pressure w.r.t. time and the dynamics of the interface, if a fixed pressure (below the saturated vapour pressure imposed on the free boundary) is imposed at the top boundary. Figure 3 shows the expected pressure values in Pascal along the geothermal reservoir ($x = 0$ is for the top level and $x = -1$ is for bottom level). We can see that the pressure field is defined on a domain growing with time. This fact is due to the adding area occupied by the vapour and then to the lowering interface as we see from Figure 4.

A first conclusion is that the evolution of the geothermal reservoir closely depends upon the pressure at the top of the reservoir, which is linked to natural events (like geysers or "fumarole" in Larderello) or the human exploitation for the production of energy (geothermal wells). Thanks to simulations we obtained an approximate way to quantify the environmental impact due to a certain exploitation.

Moreover, the results show that while the characteristic time of diffusive phenomenon is of the order of decades, as supported by geological revelations, the characteristic time of the movement of the interface is of the order of thousands of years. Thus, the time evolution of a water-dominated basin into a vapor dominated one, time within which the reservoir fluid evaporates completely, is a period that spans millennia.

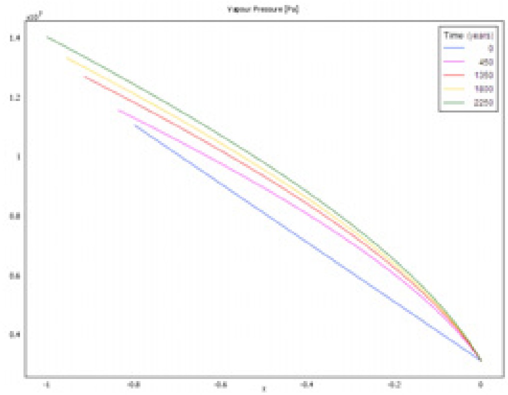

Figure 3: Pressure field in Pascal along the geothermal reservoir ($x = 0$ is for the top level and $x = -1$ is for bottom) for some times.

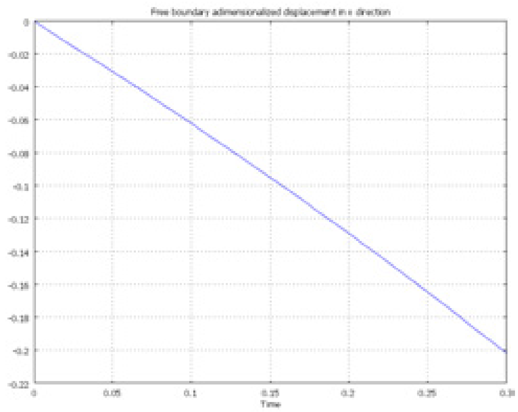

Figure 4: The interface adimensionalized displacement during the time.

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Acknowledgements

This work has been partially supported by project MAC-GEO: founded by Regione Toscana within the programme "Accordo di programma quadro ricerca e trasferimento tecnologico per il sistema produttivo".