Generation of SFQ-Pulses in SNS-Junctions: Learning Physics by COMSOL®

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- **While COMSOL does not yet have a special superconductivity module, its general mathematical module is capable of solving the time-dependent Ginzburg-Landau, or TDGL, equations. They can be justified on the basis of microscopic theory.**
- **We present simulations of the dynamics of planar SNS junctions on TDGL basis. Single-flux Abrikosov vortices launch in the DC biased junctions when the current exceed a certain critical value. It then moves along the junction until escaping from it by generating an SFQ pulse. This process is periodic in time. The reversed (also animated) process is considered when application of the SFQ pulse to the steady state under-critically DC biased junction launches an Abrikosov vortex.**
- **These results help to visualize the physical mechanisms on picosecond scale. The behavior of physical devices can be predicted and the ways of optimizing them can be found.**

TDGL equations (in dimensionless form; gapless case)**:**

$$\frac{\partial}{\partial \tau}\Psi = -\left(\frac{i}{k}\nabla + \mathrm{A}\right)^2 \Psi + \left(1 - |\Psi|^2\right)\Psi$$

Note that the expected Schrödinger equation for the Cooper pair condensate ψ is strongly modified by the residual presence of a population of normal (unpaired) electrons, so that

$i\partial\psi/\partial\tau \rightarrow \partial\psi/\partial\tau$

$$\sigma\frac{\partial A}{\partial t} = \frac{1}{2ik}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - |\Psi|^2 A - \nabla\times\nabla\times A$$

$$-\, j_n = j_s - j_{total}$$

Details and additional references can be found in our recent publications:

Phys. Lett. A **381** (2017) 2181; *ibid*., **382** (2018) 1058.

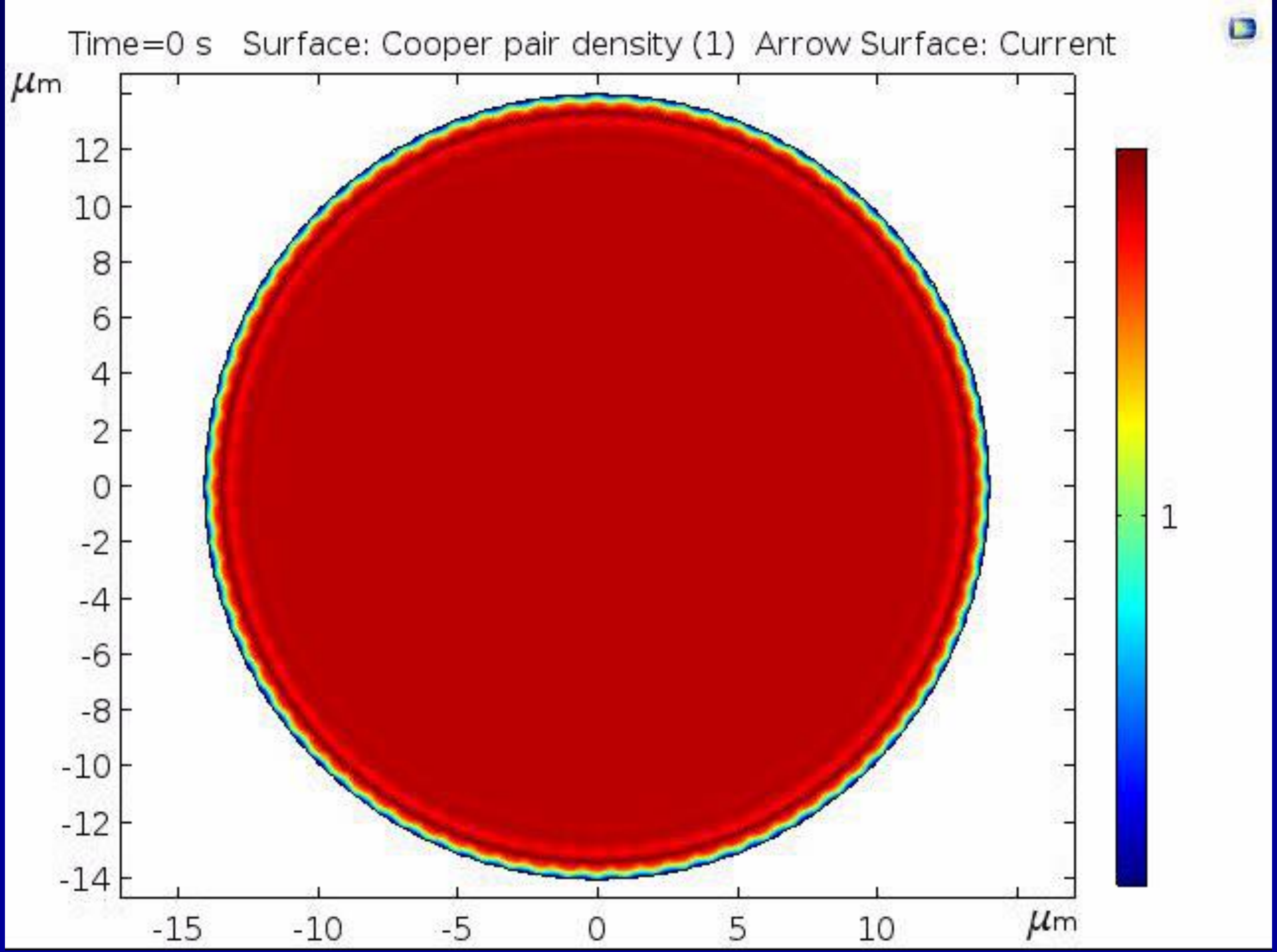
TDGL equations contain almost all the basics of superconductivity; COMSOL math module (general or coefficient form PDE) solves them easily

An example:

Meissner effect.

Type 1 superconductor (*e.g.*, k=0.4), $H<H_c$: magnetic field is fully expelled from the interior of superconductor (thin disk or long cylinder).

In this example $H=0.6H_c$.

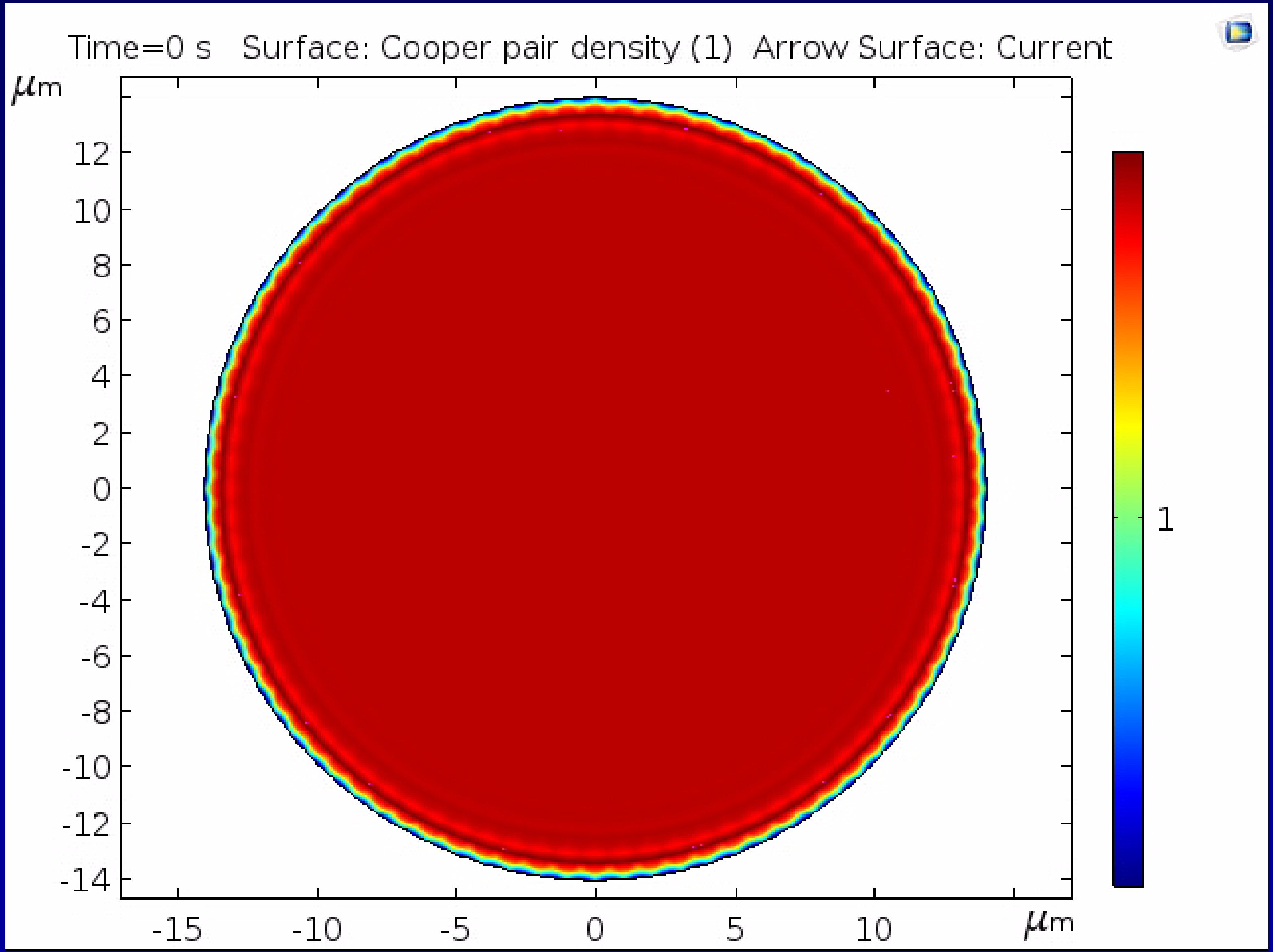


TDGL equations contain almost all the basics of superconductivity; COMSOL math module (general or coefficient form PDE) solves them easily

Another example:

If in type 1 superconductors $H>H_c$, magnetic field fully destroys the superconducting state. Experimenting with COMSOL, one can find the value of H_c.

(Here $H = 1.2H_c$ is used.)

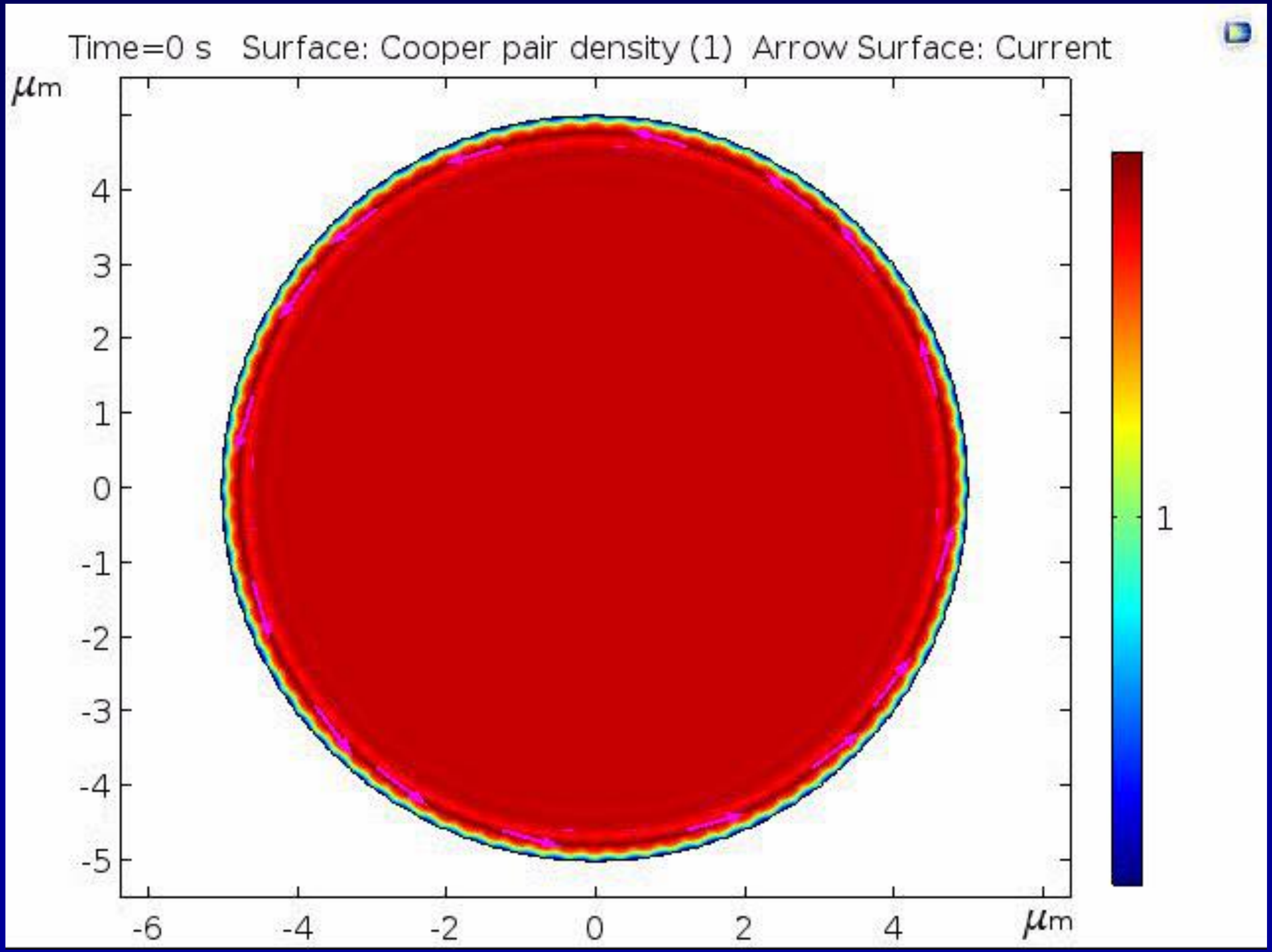


TDGL equations contain almost all the basics of superconductivity; COMSOL math module (general or coefficient form PDE) solves them easily

One more example:

In type 2 superconductors (*e.g.*, k=4), if $H<H_{c1}$, magnetic field is fully expelled from the interior of the superconductor: Meissner effect.

(In this example H is the same as in type 1 Meissner study.)
Experimenting with COMSOL one can find the critical value of k which separates type 1 and 2 superconductors.

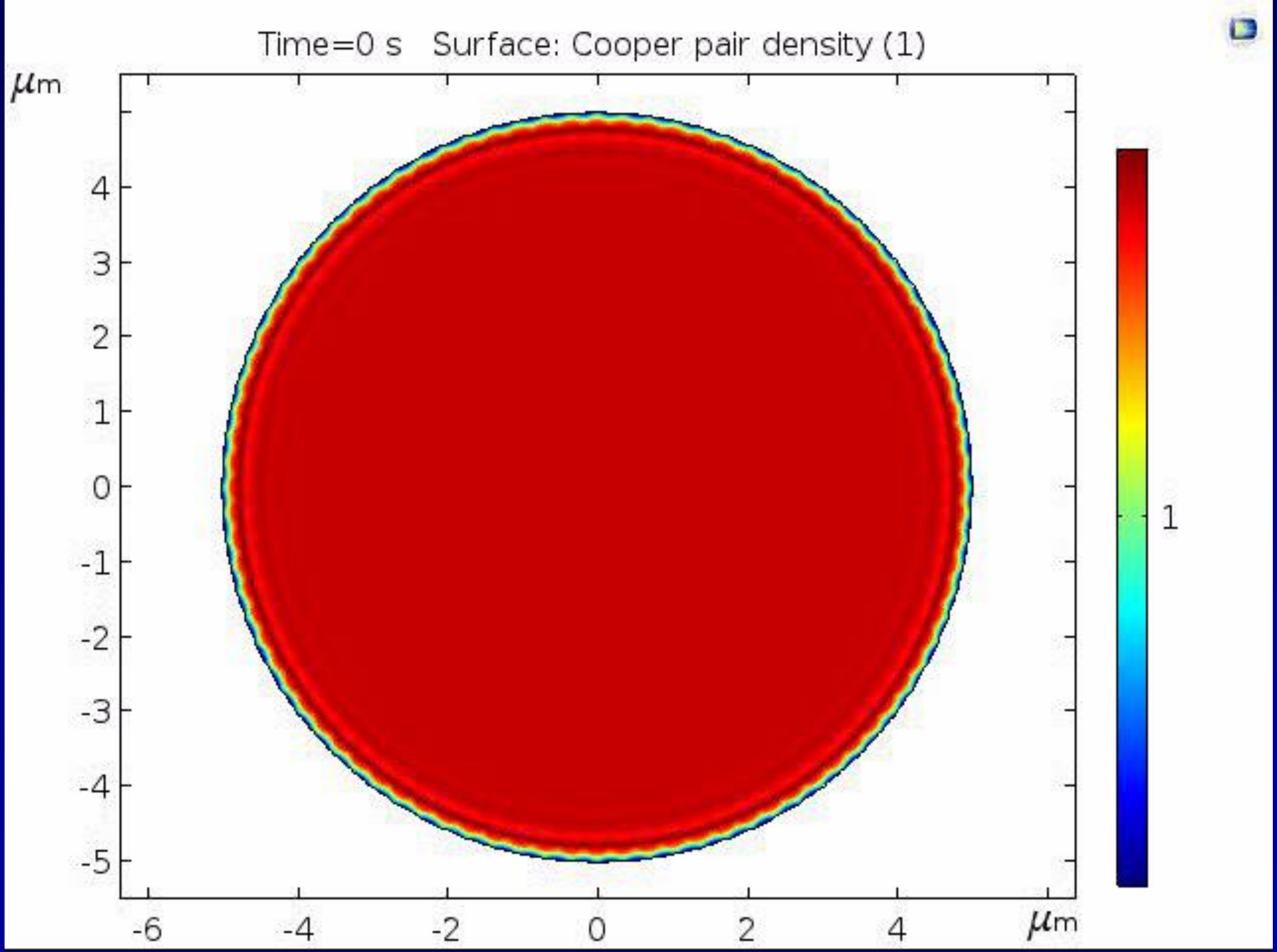


The most interesting example: Abrikosov vortices (Nobel prize in 2003)

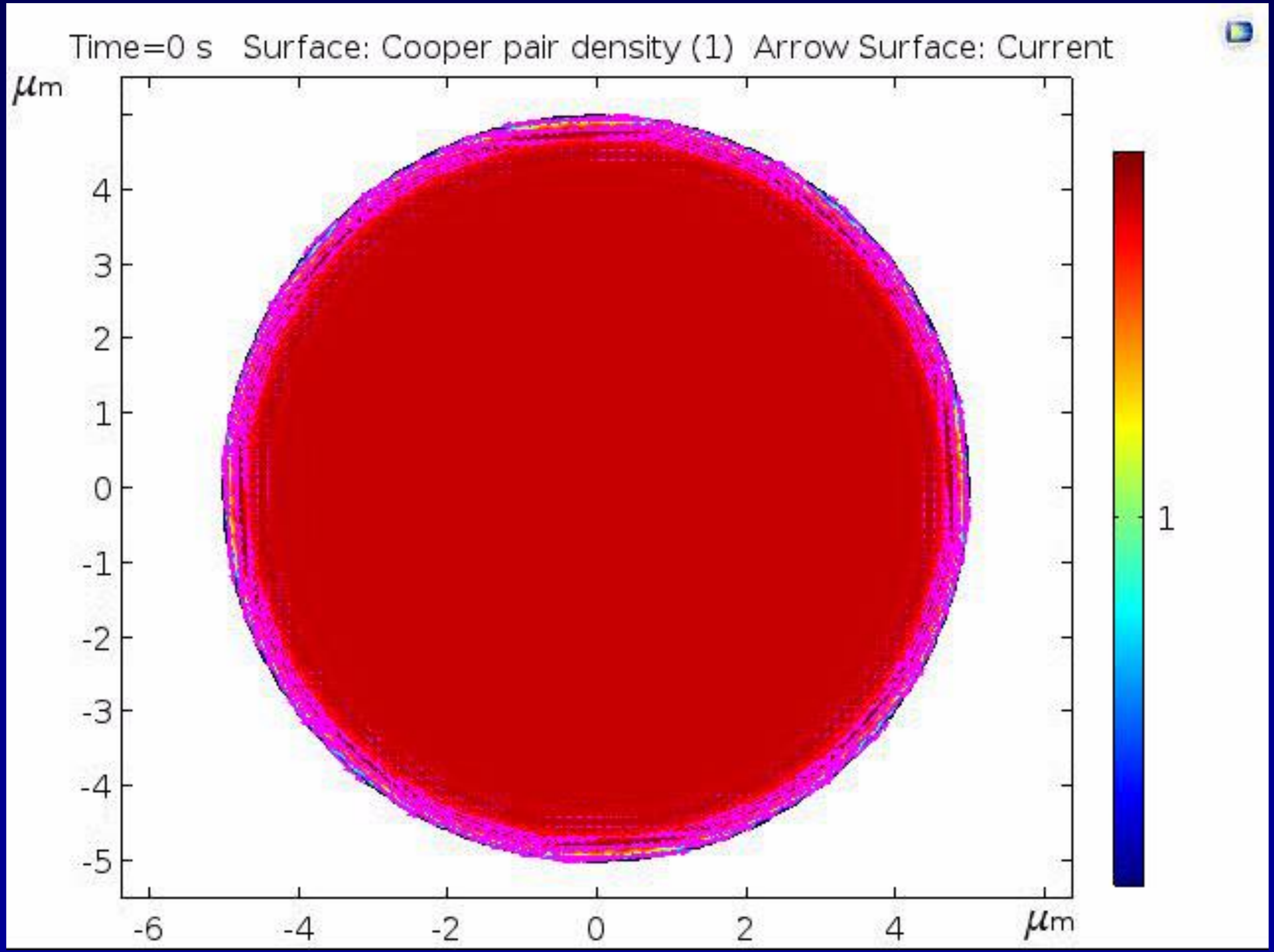
In type 2 superconductors, if $H > H_{c1}$, magnetic field penetrates into the interior in the form of single quantum vortices.

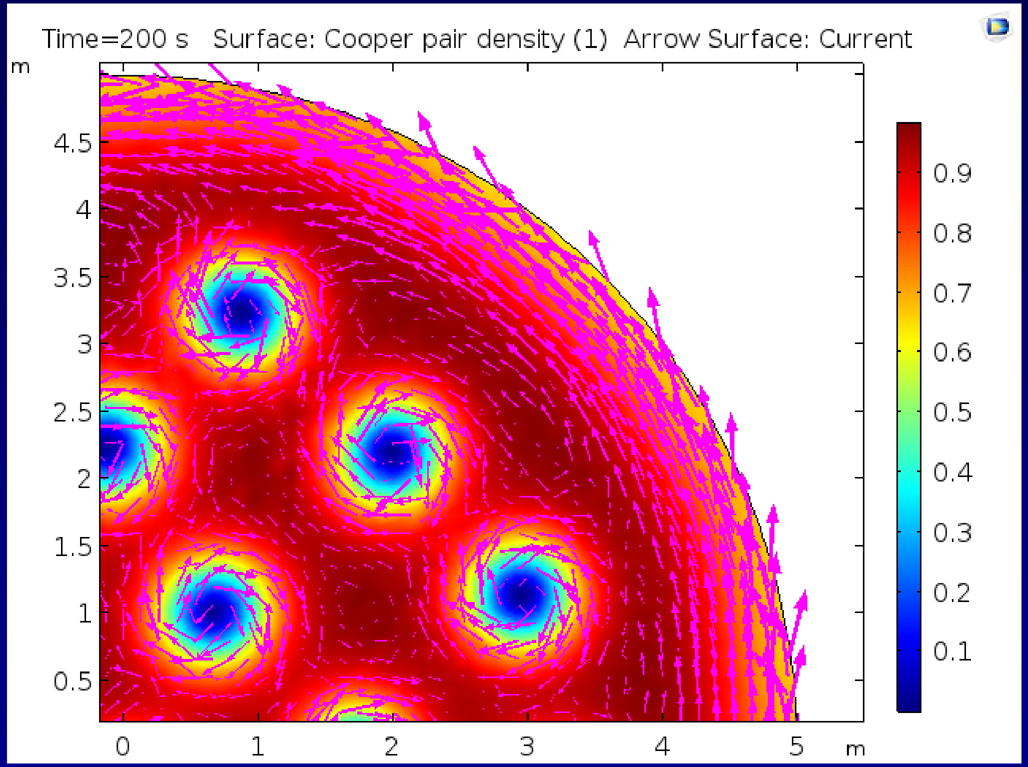
(In this example $H_{c1} < H < H_{c2}$: H=0.9) Eventual pattern demonstrates triangular lattice of vortices.

Vortices have λ_{London} characteristic size.



This COMSOL modeling allows us to reveal the microscopic structure of single-flux quantum vortices





Screening of the external field is due to the counter clockwise current. Screening of internal field is due to the clockwise current whirls.

What can we learn new?

- Superconducting Electronics employs SFQ (single-flux quantum) pulses
- They can be generated by Josephson junctions and propagate down lossless passive or active transmission lines
- They can also be regenerated by Josephson junctions
- What is their essence? “Consider them just as propagating electric pulses”
- ?!
- Clarification was very desired for novel practical applications

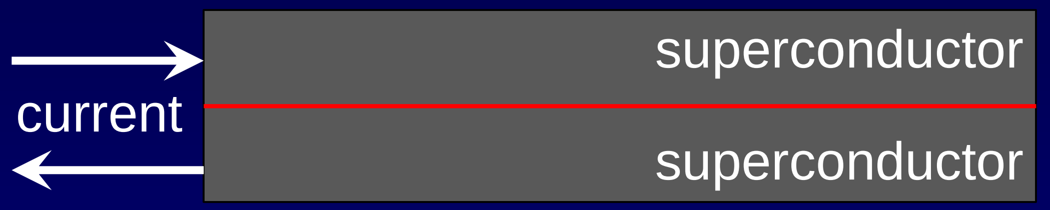
What is the Josephson junction?

- In 1962, 22 years old graduate student predicted that:

1. Without superimposed voltage, a steady current can result between two superconductors that are separated by a thin insulator.
2. If a DC voltage is added, an alternating current can result.

Nobel prize, 1973

Weakly coupled superconductors



Insulator: Angström-scale thickness

Josephson junction

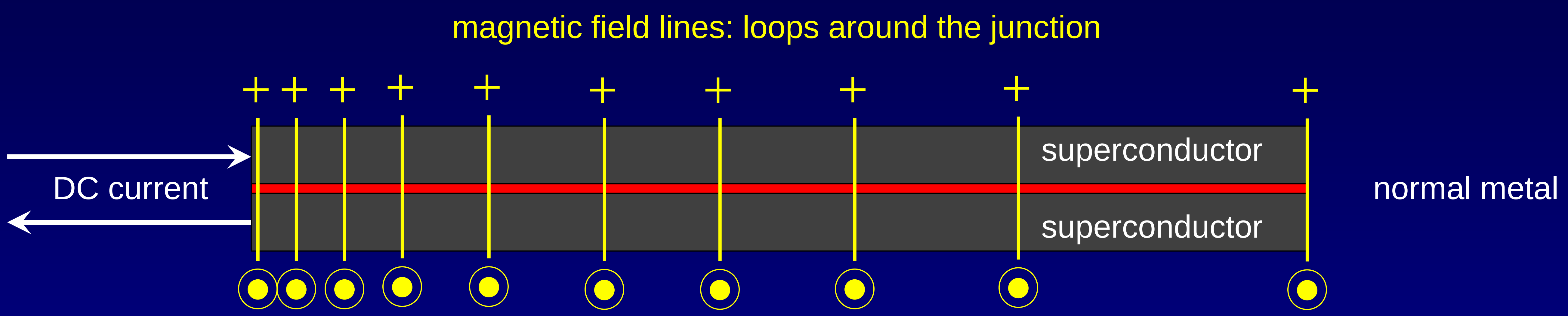
$j=j_0 \mathrm{Sin}(2eVt/\hbar+\delta\Theta)$

Normal metal: larger-scale thickness

Weakly coupled junction

Identical physics

How to model SNS junction



- While moving along the junction → super-current squeezes through the barrier/ proximitized normal metal and gradually reduces its amplitude.
- Accordingly, the intensity of the magnetic field is highest at the left end, and reduces to zero at the right end.
- For homogeneous junction parameters we assumed this behavior as linear along the SNS:

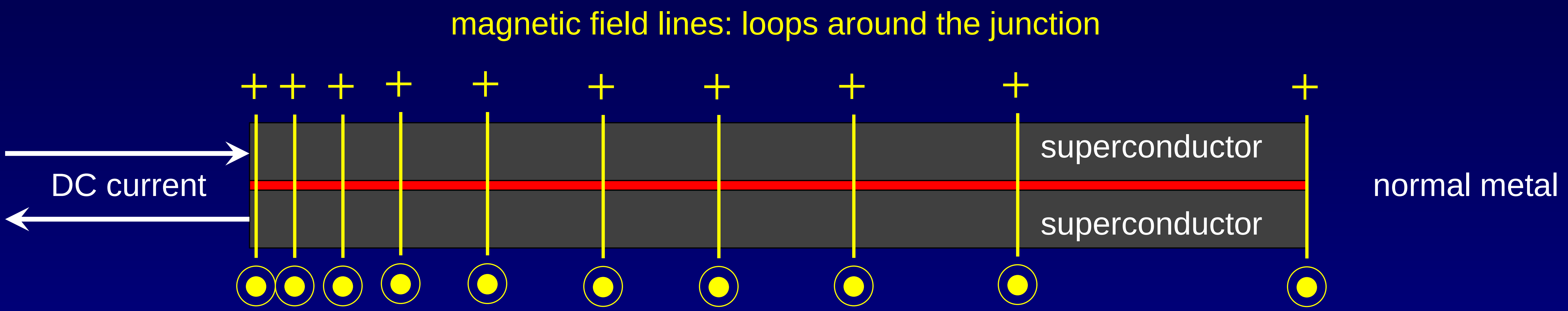
  $H_z = H_0(L/2-x)/L$ on the top boundary line

  $H_z = -H_0(L/2-x)/L$ on the bottom boundary line

here L is the length of the junction along the current flow (x-axis), the coordinate origin is in the middle of the junction.

- Thus, current is being determined via boundary conditions on H_z (z is orthogonal to the picture plane).

Modeling SNS junction in TDGL

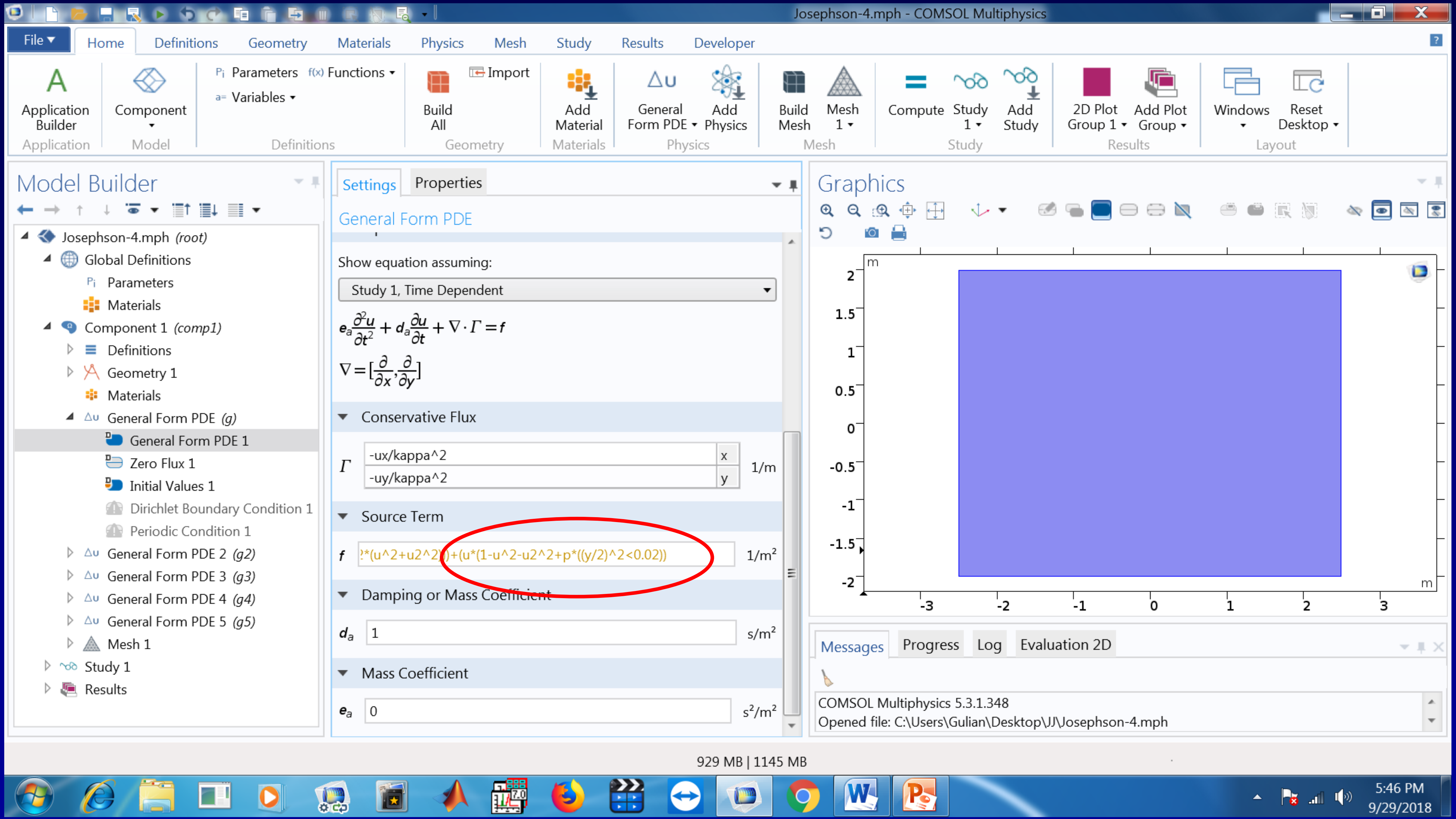


Inclusion of the "weak coupling" via the p-term:

$$\frac{\partial}{\partial \tau}\Psi = -\left(\frac{i}{k}\nabla + \mathrm{A}\right)^2 \Psi + \left(1 - |\Psi|^2 + p\right)\Psi$$

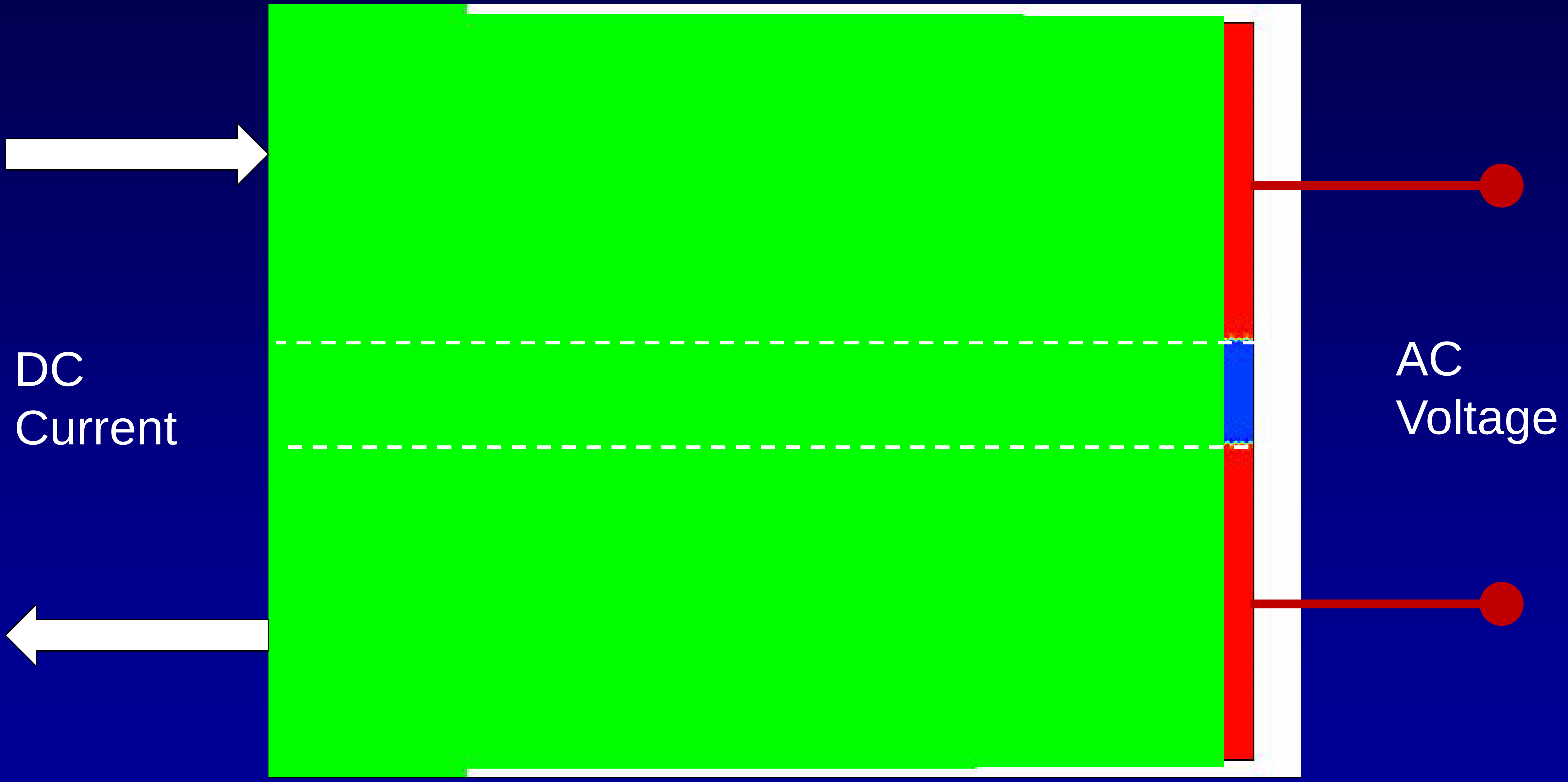
Negative values of p=p(y) in our case reduce locally critical temperature mimicking proximitized N-layer (p=–0.3 was chosen at calculations).

Implementation of proximitized N-layer in COMSOL



Shown is the equation for the real part of the ψ-function ("u" in COMSOL); the same entry is in the equation for the imaginary part of ψ-function (i.e., "u2").

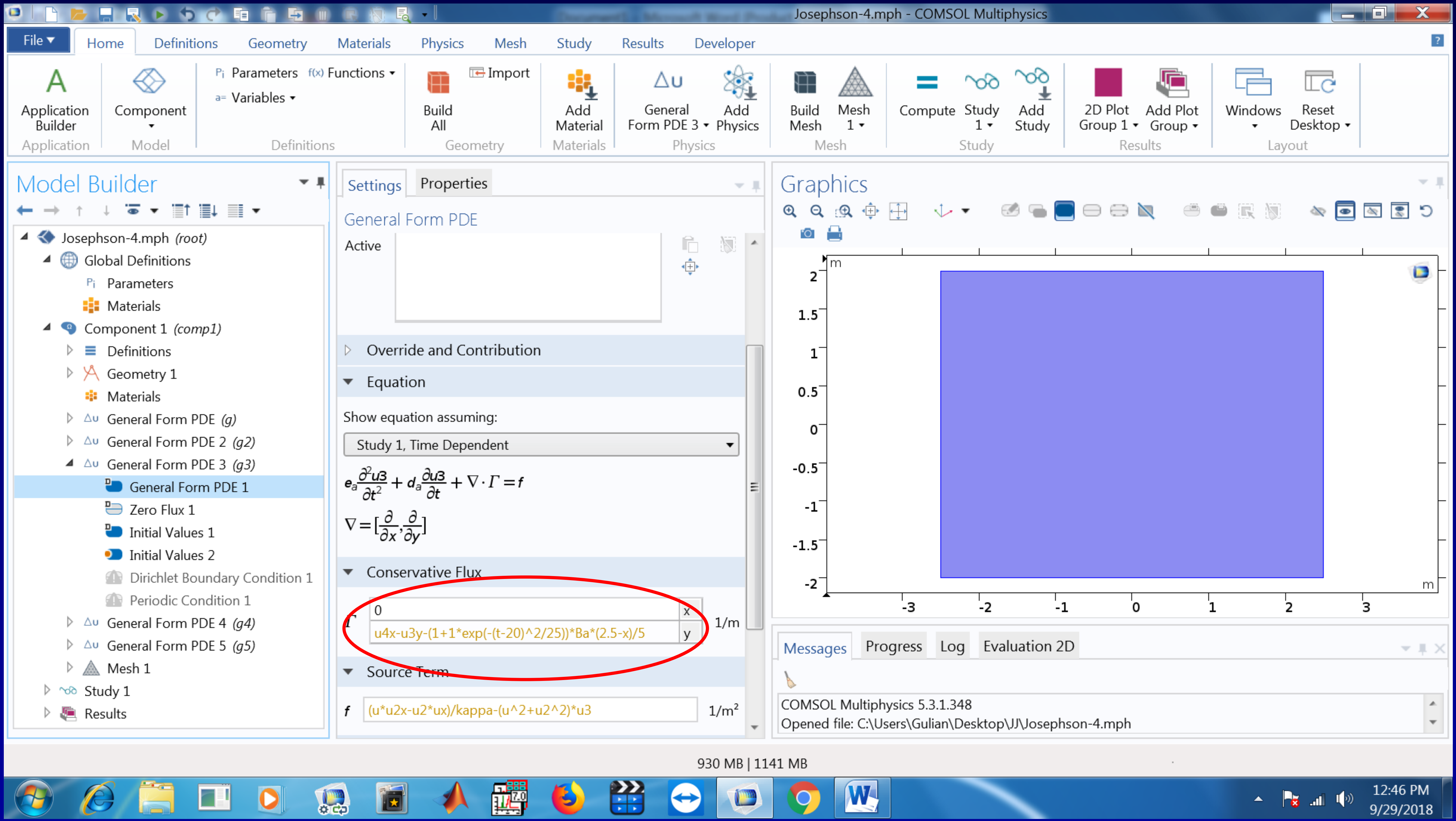
Resulting picture: AC pulse generator



Green arrows are the electric field vector

Generating an SFQ-pulse

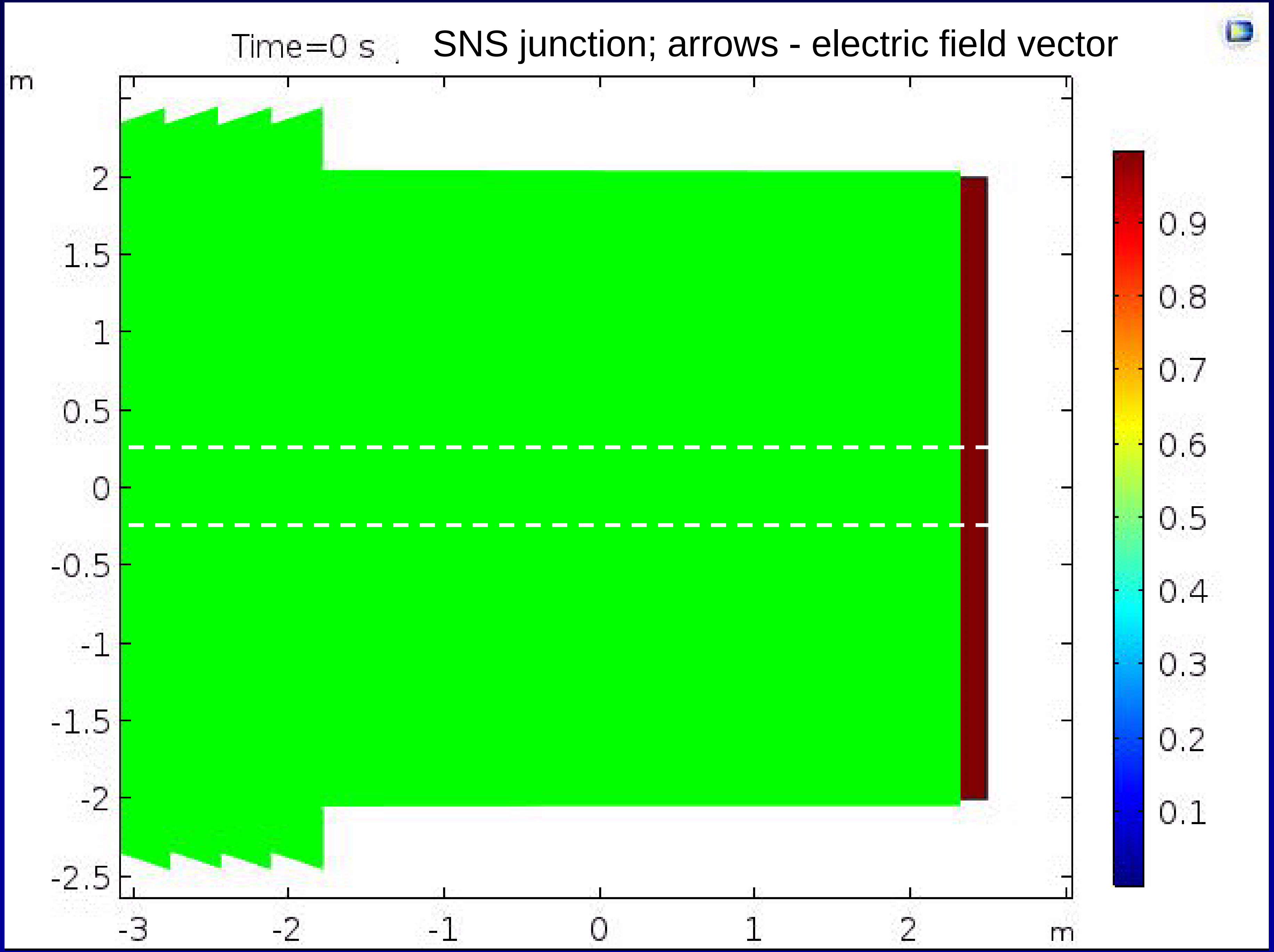
DC biased SNS-junction is in undercritical regime. A short voltage pulse (an SFQ pulse) is being absorbed, temporarily moving SNS into overcritical regime, thus generating new SFQ pulse.



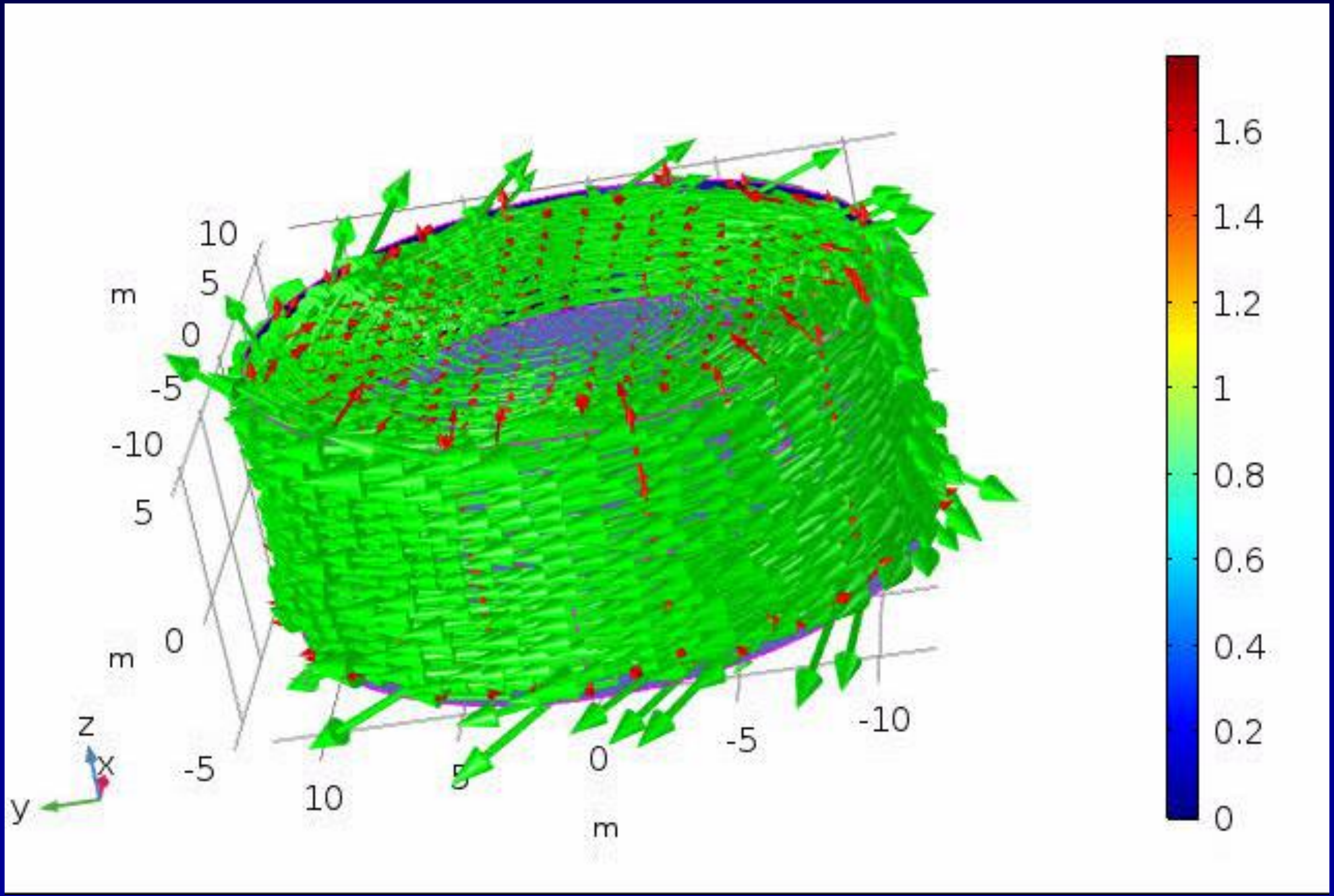
Visualization of the SFQ-pulse dynamics (picosecond scale)

t=0: DC biasing the SNS junction into the undercritical regime.

Subsequent absorption of the short SFQ pulse creates a larger current, temporarily moving SNS into overcritical regime, originating an Abrikosov vortex. It propagates along the junction and generates a new SFQ pulse when leaving it.



Shortcut to Superconductivity via COMSOL



Using TDGL + COMSOL the basics of super-conductivity can be understood in a very convenient, time-saving and productive way.

More examples will come soon.

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Thank you for your attention!