Relativistic Quantum Mechanical Wave Functions for Fermion Particles in Electric or Magnetic Fields

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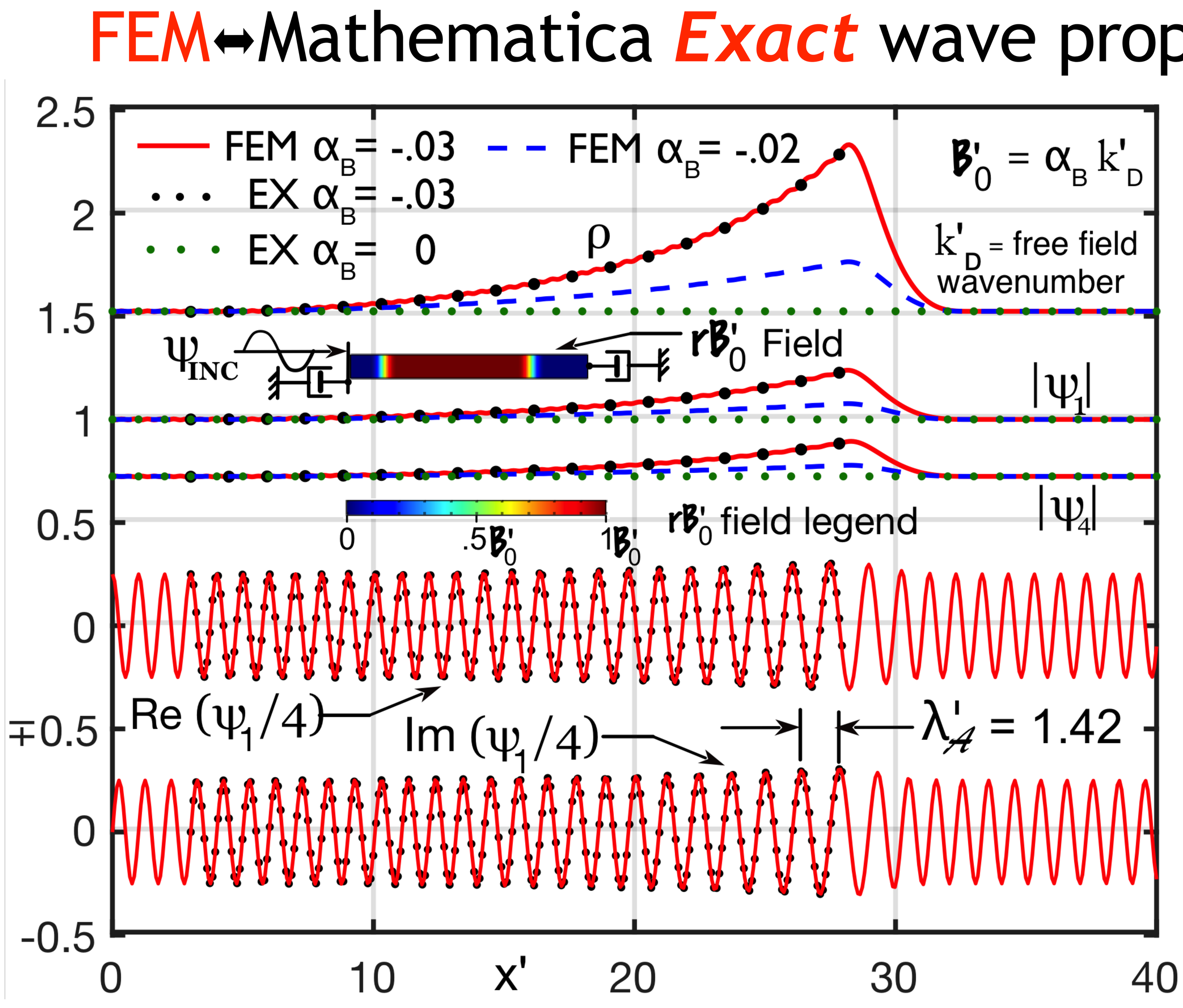
Introduction: Find the relativistic quantum mechanics steady state wave function $\Psi_m(x,y,z,\omega)$ as a solution to the Dirac equations with pre-existing magnetic and electric potentials $\bar{A}$, ϕ. The probability density, ρ, of a particle's location is given by $\rho=\sum|\Psi_m|^2$ m=1..4

Computational Method: The EM Dirac equations [1] for the behavior of a particle of mass *m* with $M=mc/\hbar$, c=light speed, $\hbar$=Planck's constant, $\bar{\mathbf{A}}=\bar{A}e/\hbar$, $\Phi=e\phi/c\hbar$, e=charge: are solved with COMSOL'S *"Coefficient-Form PDE"*.

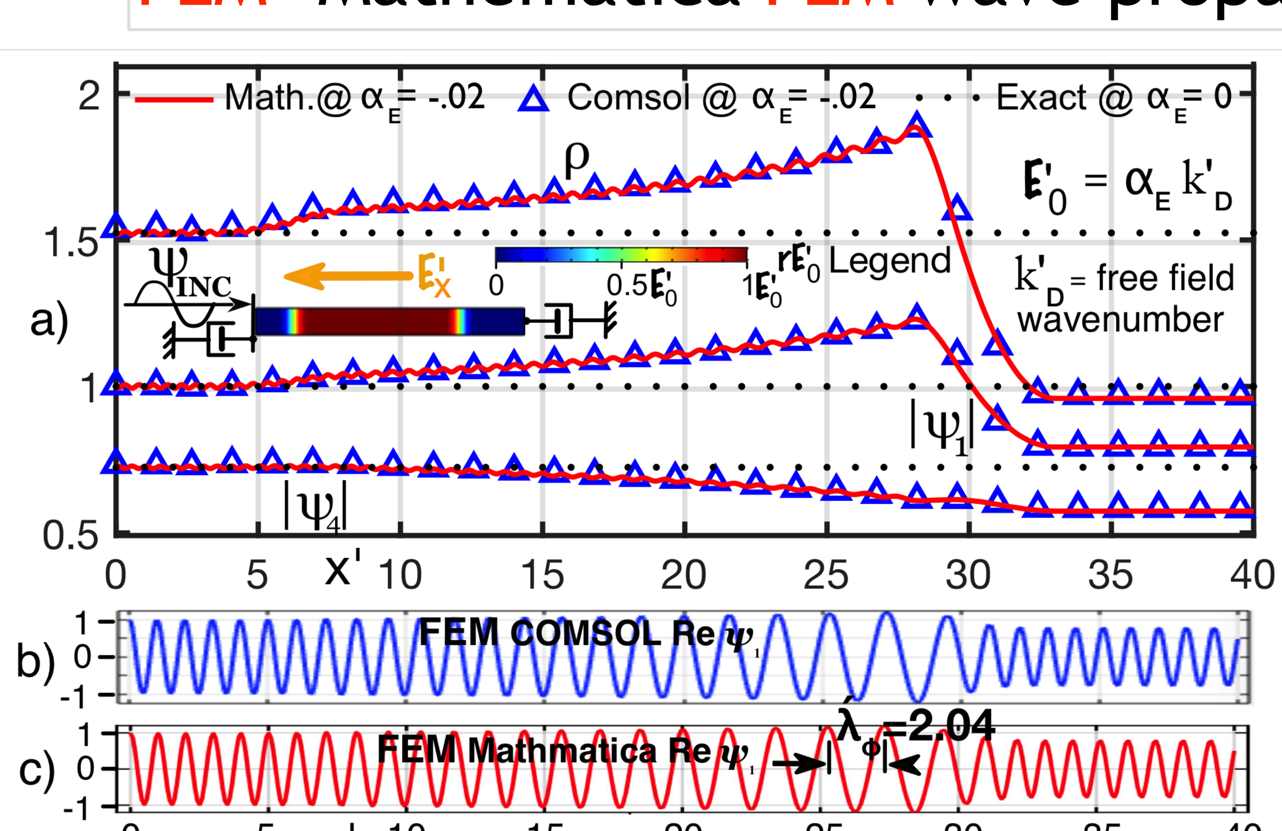
$$\frac{1}{c}\frac{\partial\Psi_1}{\partial t}+\frac{\partial\Psi_4}{\partial x}-i\frac{\partial\Psi_4}{\partial y}+\frac{\partial\Psi_3}{\partial z}+i\Psi_1(\Phi+M)+i(i\mathbf{A}_y\Psi_4-\mathbf{A}_z\Psi_3-\mathbf{A}_x\Psi_4)=0$$
$$\frac{1}{c}\frac{\partial\Psi_2}{\partial t}+\frac{\partial\Psi_3}{\partial x}+i\frac{\partial\Psi_3}{\partial y}-\frac{\partial\Psi_4}{\partial z}+i\Psi_2(\Phi+M)+i(A_z\Psi_4-\mathbf{A}_x\Psi_3-i\mathbf{A}_y\Psi_3)=0 \quad (1)$$
$$\frac{1}{c}\frac{\partial\Psi_3}{\partial t}+\frac{\partial\Psi_2}{\partial x}-i\frac{\partial\Psi_2}{\partial y}+\frac{\partial\Psi_1}{\partial z}+i\Psi_3(\Phi-M)+i(i\mathbf{A}_y\Psi_2-\mathbf{A}_z\Psi_1-\mathbf{A}_x\Psi_2)=0$$
$$\frac{1}{c}\frac{\partial\Psi_4}{\partial t}+\frac{\partial\Psi_1}{\partial x}+i\frac{\partial\Psi_1}{\partial y}-\frac{\partial\Psi_2}{\partial z}+i\Psi_4(\Phi-M)+i(\mathbf{A}_z\Psi_2-\mathbf{A}_x\Psi_1-i\mathbf{A}_y\Psi_1)=0$$

When the wave vector $\bar{\mathbf{k}}$ is in the xy plane, $\partial\Psi_m/\partial z$ terms drop out and the 1st & 4th eqs. decouple, where Ψ_1,Ψ_4 are solved alone.

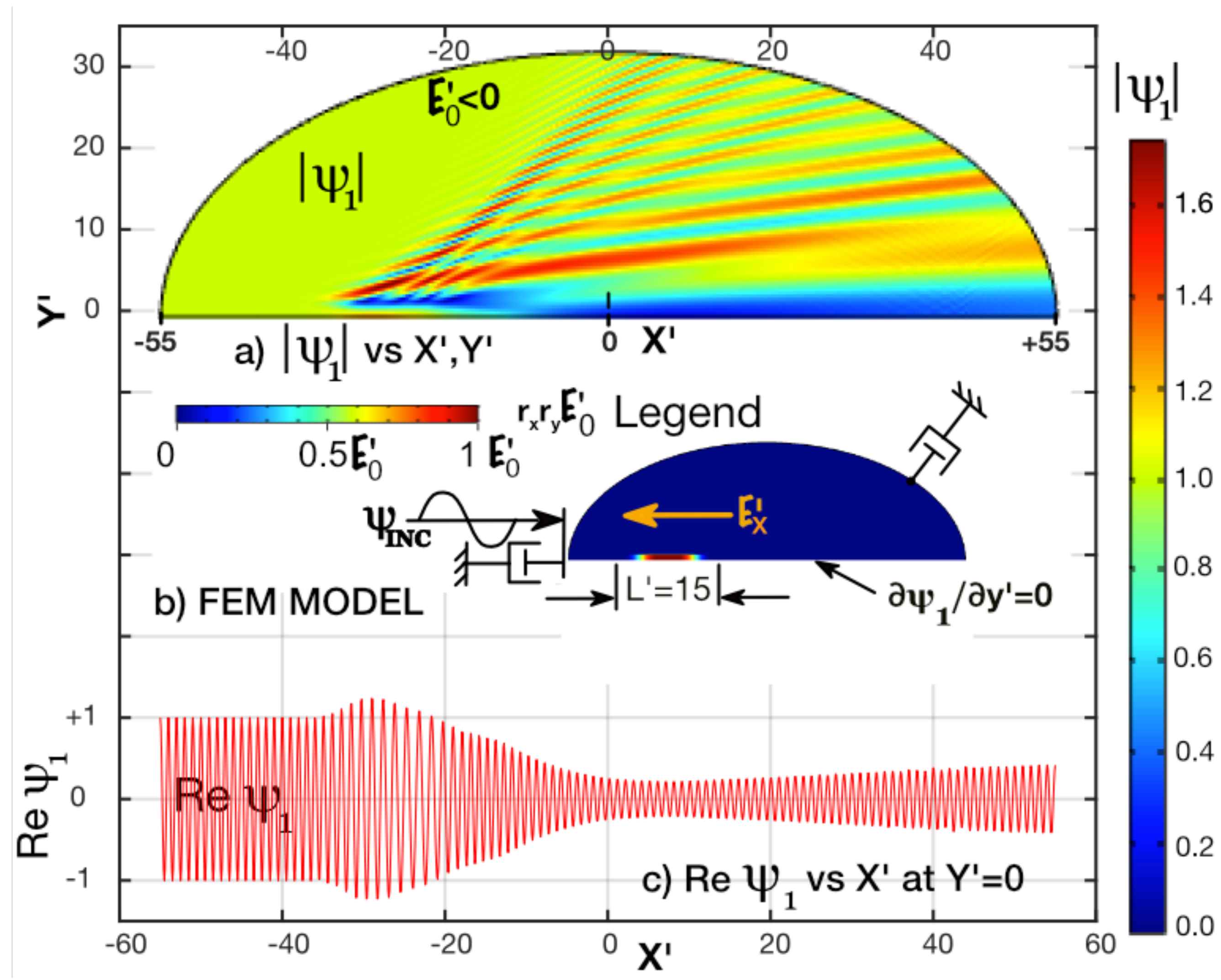
Results: • **Fig.1** <u>*Magnetic B Field On*</u> below validates the $\Psi_1=1e^{-i\omega' t'}$ end driven *Wave Guide* PW COMSOL FEM↔Mathematica ***Exact*** wave propagation vs $x'=x/\lambda_D$ and is shown for 3 values of magnetic field strength parameter α_B={.0, -0.02,-0.03}.The magnetic B' field effect gradually increases the λ'_A spatial wave length and ρ probability density vs +x'.



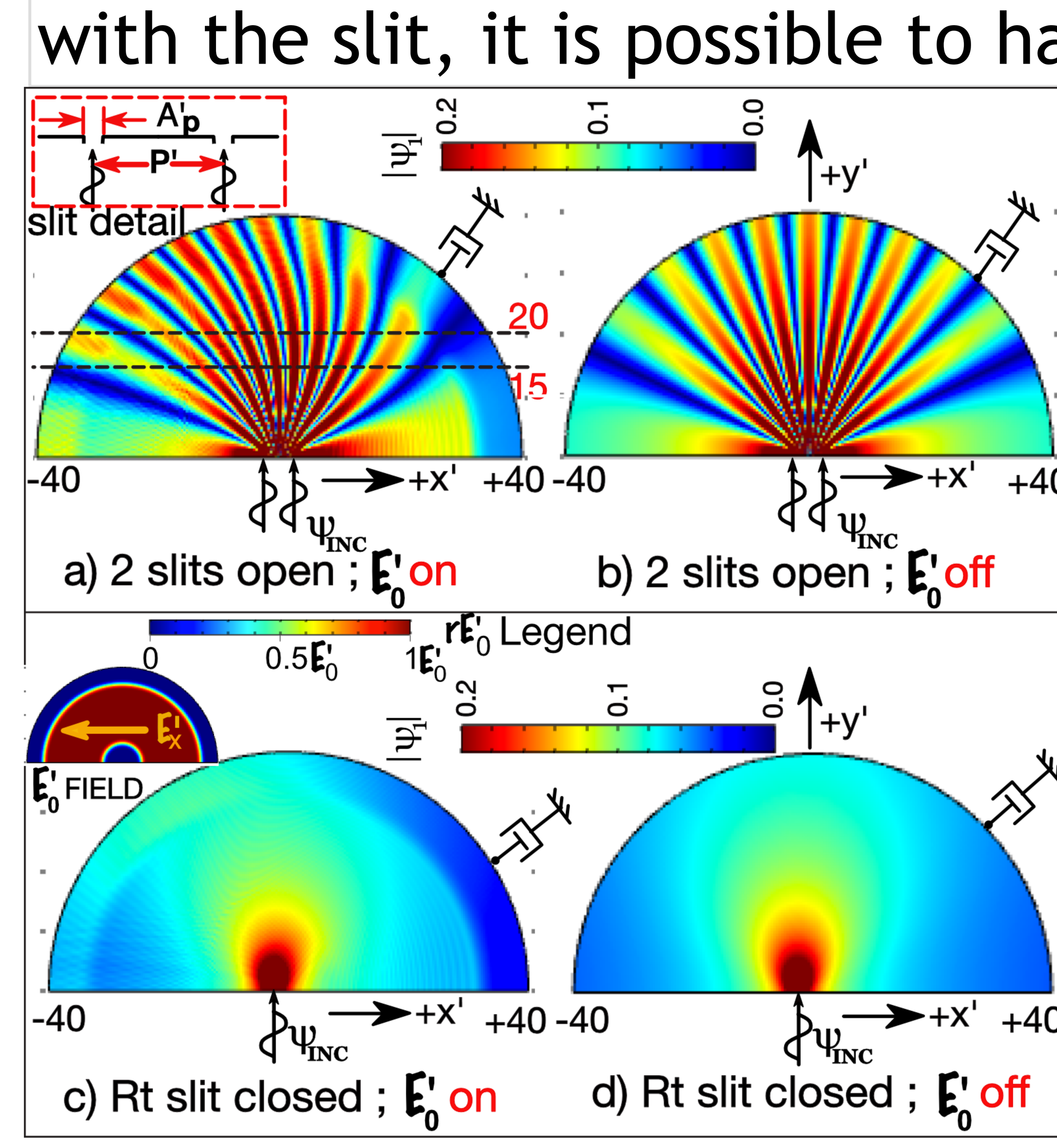
• **Fig.2** <u>*Electric E′ Field On*</u> below validates the $\Psi_1=1e^{-i\omega' t'}$ end driven *Wave Guide* PW COMSOL FEM↔Mathematica FEM wave propagation vs $x'=x/\lambda_D$ and is shown for 2 values of electric field strength parameter α_E={.0, -0.02}. The electric E' field effect gradually increases the λ'_ϕ spatial wave length and ρ probability density vs +x'. Cutoff functions are r, r_x, r_y .



• **Fig.3** <u>*Electric E′ Field On*</u> upper right is like Fig. 2 case (except roof of wave guide removed) where a L′=15 x W′=4 finite E' field (α_E=-0.02) is embedded in a free field and subject to an incident $\Psi_1=1e^{-i(x'k'_D-\omega' t')}$ PW. Unlike the Fig.2 waveguide, due to diffraction, the Fig.3c downstream $\Psi_1(x')$ field slowly builds back up towards the incident 1.0 free field value. Again, the Ψ_1 spatial wavelength gradually increases while passing thru the E' field & slowly reverts→1 downstream.



• **Fig.4** <u>*2 Slit Demo; Electric E′ Field On vs Off*</u> Particles fired at 2 slits, is a classic quantum mechanics demo, represented by a $\Psi_1=1e^{-i(x'k'_D-\omega' t')}$ PW wave function incident upon the slits. Figs.4a and 4b compare bands of $|\Psi_1|$constructive and destructive interference with the E' field on vs. off. The effect of the E' field (with electric field strength parameter α_E=-0.02) is to curve the fan blade like bands compared to the straight bands with the E' field turned off. Due to the angular shape of the interference bands, at some points directly in line with the slit, it is possible to have a lower location probability than a corresponding shifted point off the slit line. Closing the right slit, the interference patterns are gone as seen in Figs. 4c & 4d where the probability is highest inline with the slit.



Conclusions: The *Coefficient-Form PDE* option successfully validated the EM time independent Dirac equation solutions. In the 2 slit demo, banded downfield groupings of particle locations, as inferred by (4b), are also observed experimentally.

References:1. P. Strange, Relativistic <u>Quantum Mech.</u>, Camb. Univ. Press 1998

Excerpt from the Proceedings of the 2018 COMSOL Conference in Boston