

COMSOL Conference – Lausanne – October 2018

Nanoscale Heat Transport and Phonon Hydrodynamics

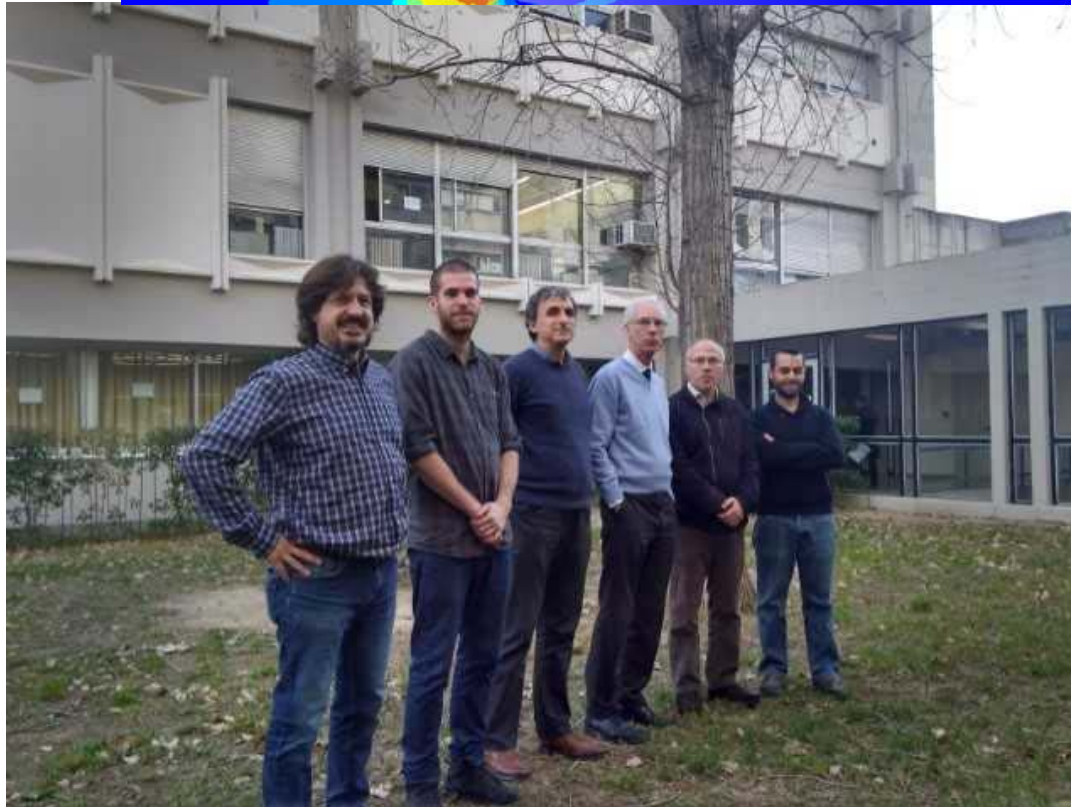
A. Beardo, L. Sendra, J. Bafaluy, J. Camacho, F. X. Alvarez

Universitat Autònoma de Barcelona
Physics Department
albert.beardo@uab.cat

UAB

Universitat Autònoma de Barcelona

**COMSOL
CONFERENCE
2018 LAUSANNE**



PHYSICS DEPT.
F. Xavier Alvarez
Albert Beardo
Javier Bafaluy
David Jou
Juan Camacho
Lluc Sendra

ELCTR. ENG. DEPT.
Xavier Cartoixà

MATH DEPT.
Marc Calvo
Tim Mayers
Mark Hernessey

PURUE UNIVERSITY
Ali Shakouri
Amirkoushar Ziabari

ICMAB
Riccardo Rurali
Pol Torres

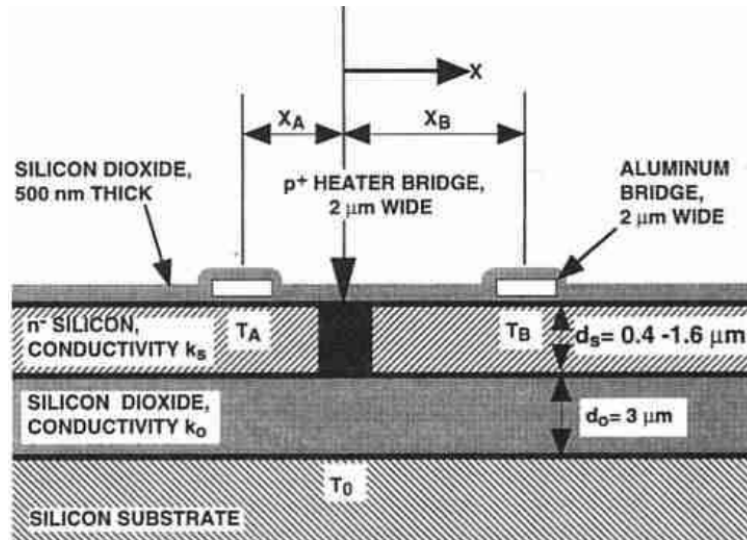
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- Motivation
- Nanoscale Heat Transport
- Kinetic Collective Model
- Experimental validation
- Conclusions

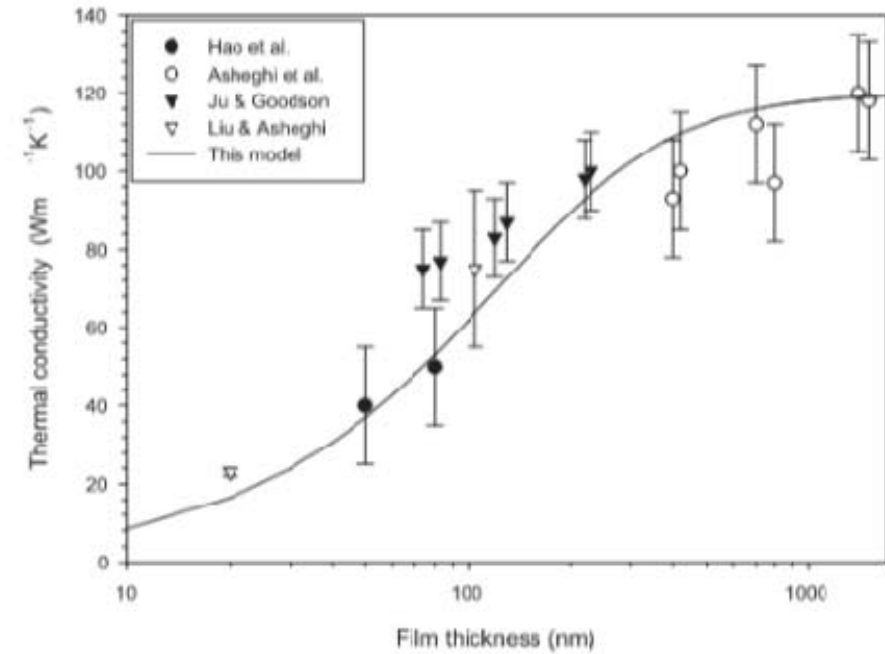
Fourier / Diffusive model

$$c \frac{dT}{dt} = -\nabla \cdot q$$

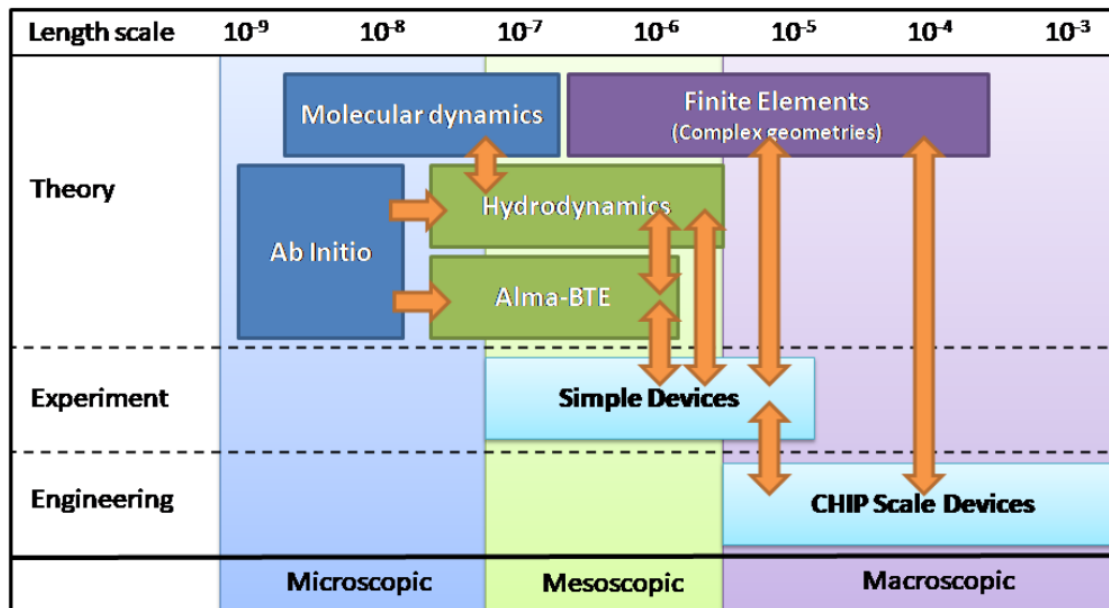
$$q = -\kappa \nabla T$$



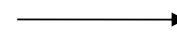
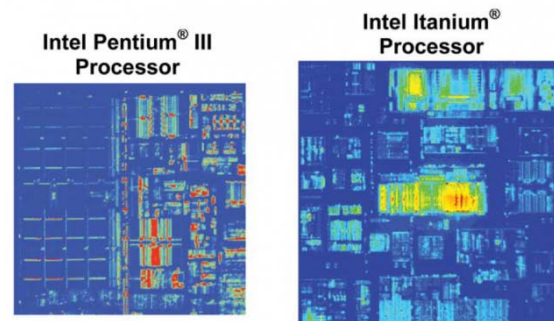
Breakdown of Fourier law at reduced length and time scales.



3>> Motivation



- * Obtain an improved equation of transport derived from the **microscopic description of the phonons dynamics**.
- * Improve the thermal management in nano scale semiconductor devices.



Predict the appearance of hot spots.

Boltzmann Transport Equation

Describes the non equilibrium evolution of the **phonons** (energy carriers).



BTE output

Phonon distribution function $f(\kappa, x, t)$



Moment description

$$M_i(x, t) = \int \kappa \dots \kappa f(\kappa, x, t)$$



Grad/Chapman-Enskog equations
Guyer-Krumhansl equations



Thermodynamic output

Heat Flux \mathbf{q}

Temperature T

Hydrodynamic Equations

$$c \frac{dT}{dt} + \nabla \cdot \mathbf{q} = 0$$

$$\tau \frac{\partial \mathbf{q}}{\partial t} + \mathbf{q} + \kappa \nabla T = \ell^2 (\nabla^2 \mathbf{q} + 2 \nabla \nabla \cdot \mathbf{q})$$

Relaxation
Time

Bulk Thermal
conductivity

Non Local
Length

Slip Boundary Conditions

$$\mathbf{q}_t = -Cl \nabla \mathbf{q}_t \cdot \mathbf{n}$$

$$\mathbf{q} \cdot \mathbf{n} = 0$$

Surface
specularity

- Calculated from first principles.
- Material properties.
- Only depend on temperature.

6 >> Kinetic Collective Model

COMSOL interface for solving thermal transport in complex geometries.

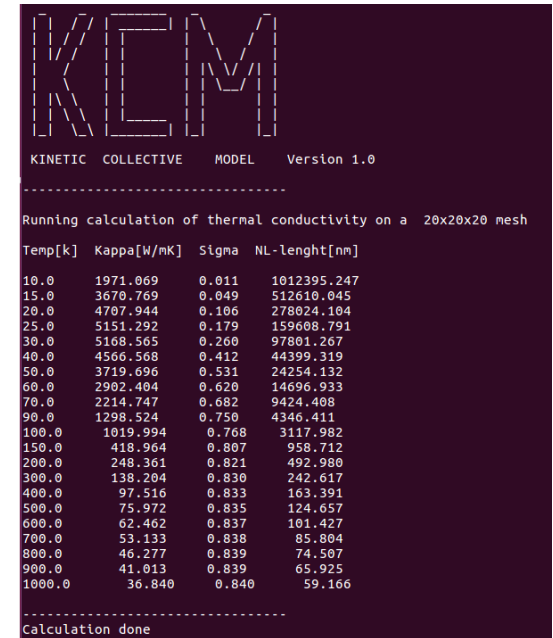
The screenshot shows the COMSOL interface for the 'Hydrodynamic Heat Transfer 1' study. The left sidebar shows the model tree with 'Nanoscale Heat Transfer - Kinetic Collective Model (kcm)' selected. The main window displays the following settings:

- Label:** Hydrodynamic Heat Transfer 1
- Domain Selection:** Manual
- Active:** 1, 2
- Equation:** Study KCM, Time Dependent
- Equation:**

$$C_v \frac{\partial T}{\partial t} + \nabla \cdot \mathbf{q} = Q$$

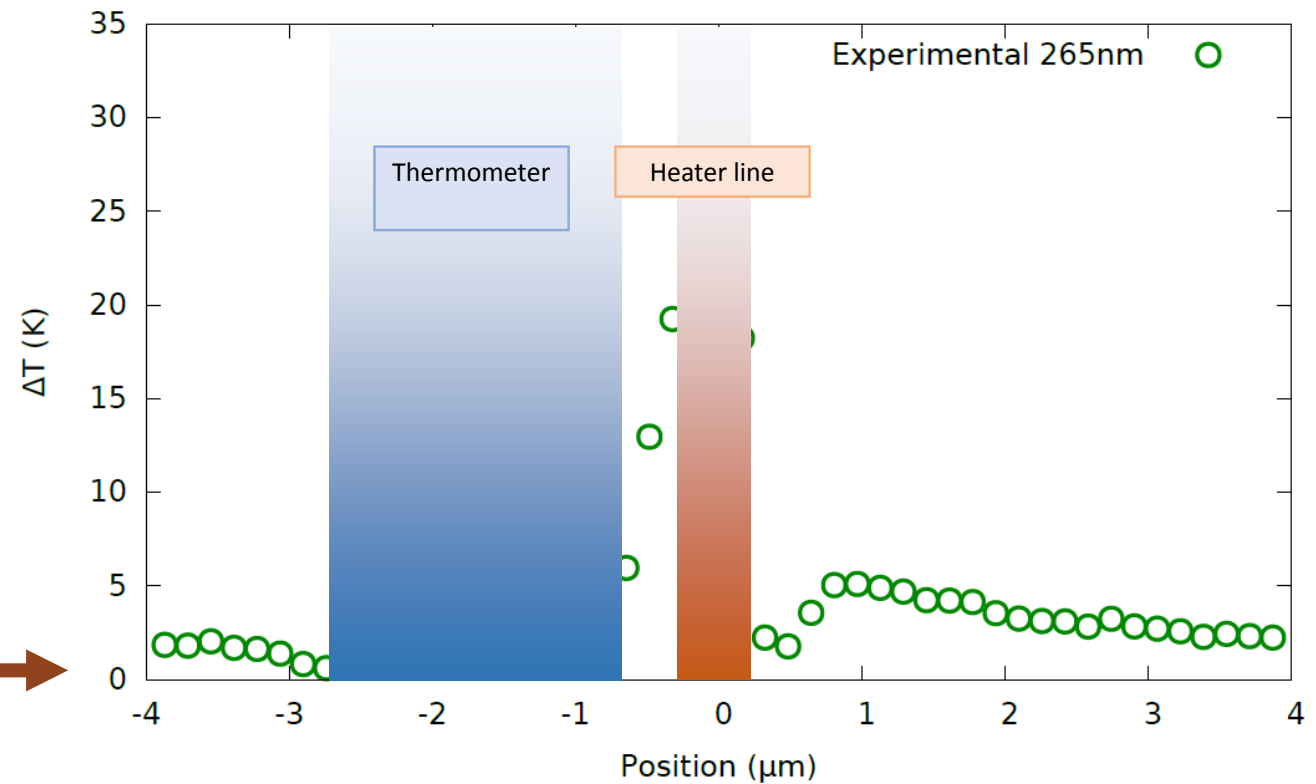
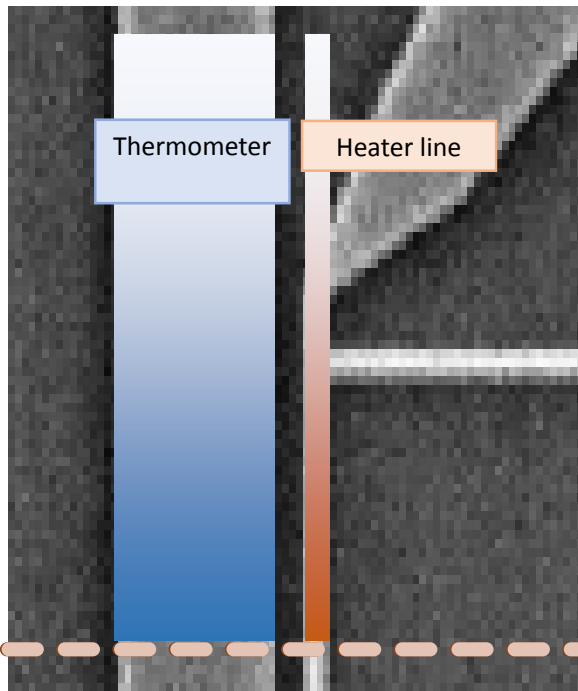
$$\mathbf{q} + \tau \frac{\partial \mathbf{q}}{\partial t} + k \nabla T = l^2 (\nabla^2 \mathbf{q} + 2 \nabla \nabla \cdot \mathbf{q})$$
- Heat Transfer Parameters:**
 - Volumetric heat capacity: C_v [J/(m³·K)]
 - Thermal conductivity: k [W/(m·K)]
 - Hydrodynamic Length: l [m]
 - Hydrodynamic Time: τ [50e-12[s]]

First principles calculation of κ, l, τ



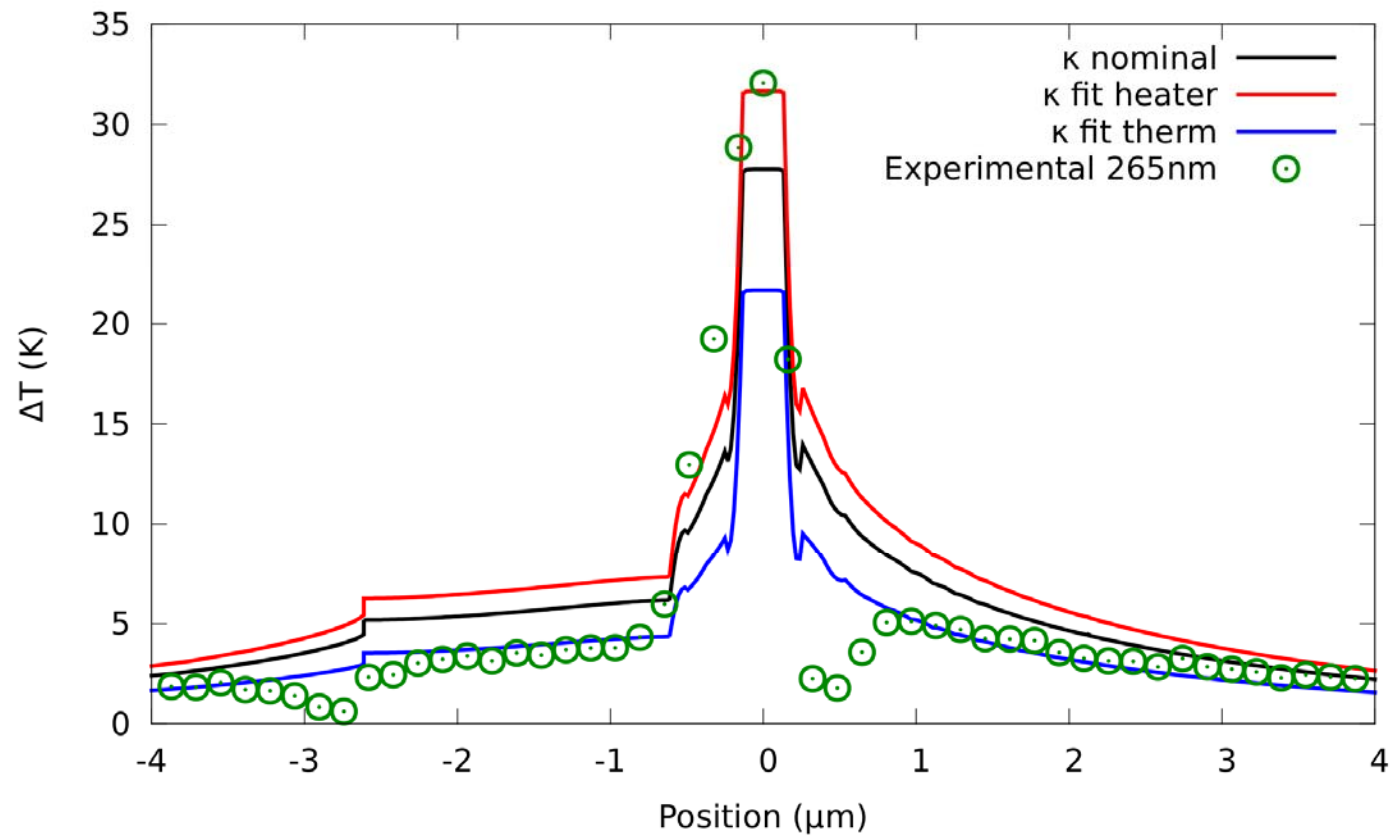
- Discontinuous Galerkin implementation of the boundary conditions.
- Stabilization.
- Multiphysics coupling with Fourier domains (metal domains).

Heater on InGaAs substrate.



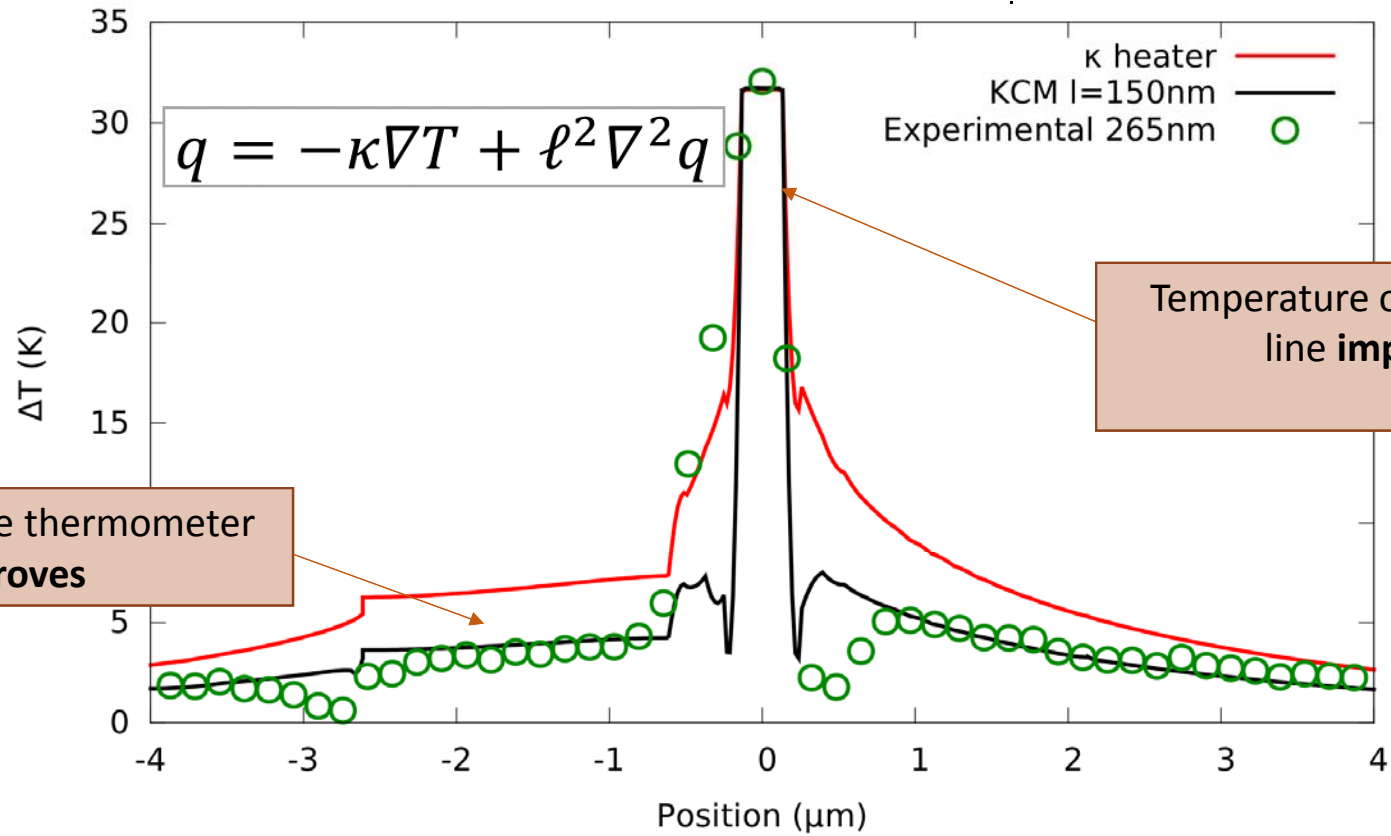
We obtain a thermal map of the surface of the sample using the optical setup. Heater line and thermometer are also obtained using electrical measurements.

8>> Experimental validation



Fourier's law cannot describe thermal transport in this setup. New equation is needed.

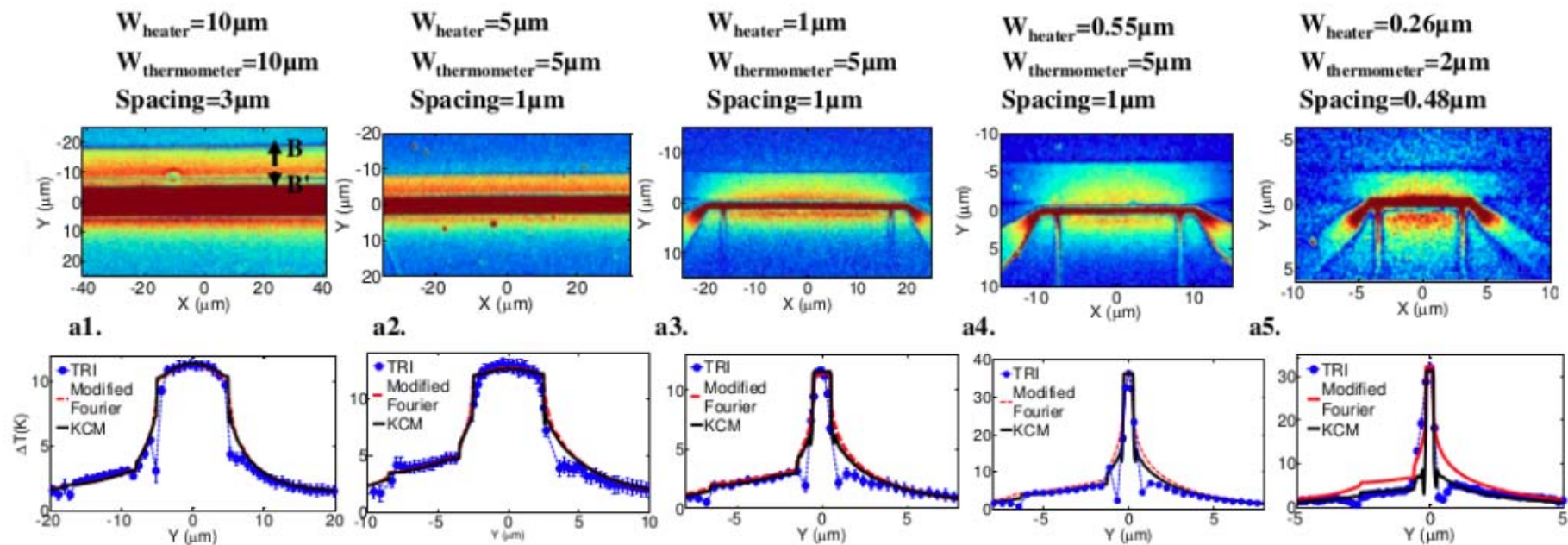
9>> Experimental validation



Kinetic Collective Model performs better to experimental data

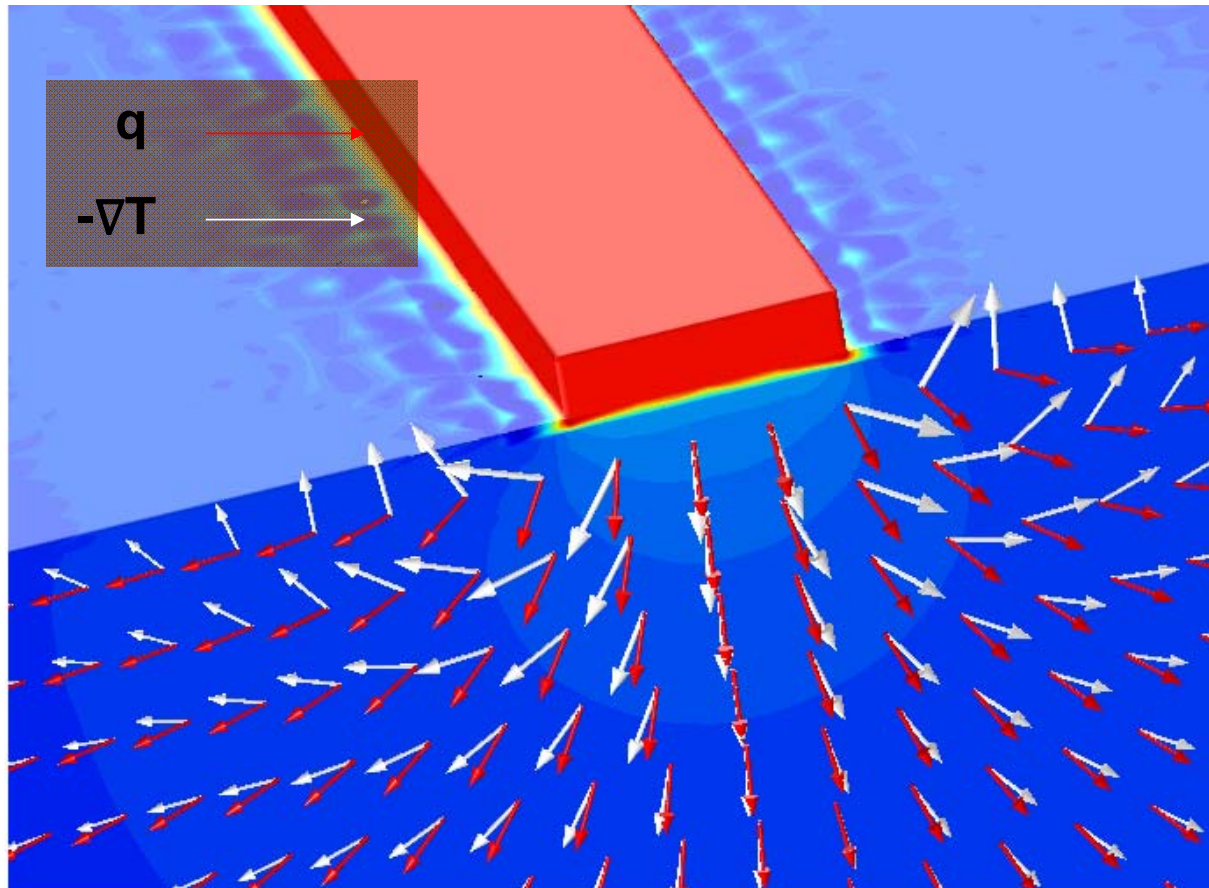
Ziabari et al., Nat. Commun. **9**, 255 (2018)

- At large sizes we recover Fourier model.
 - The smaller the size, the larger the effect.
- At extremely small sizes, the model fails due ballistic effects.



11 >> Experimental validation

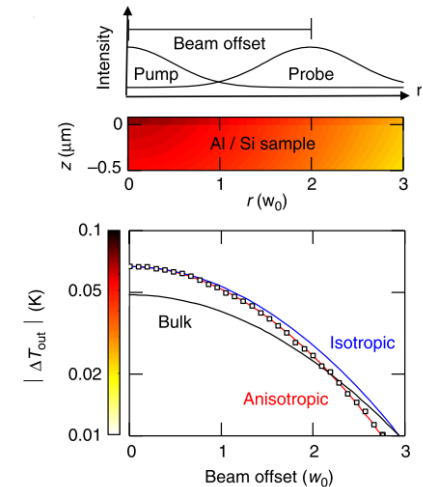
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ANISOTROPIC BEHAVIOUR

Temperature gradient and heat flux are not parallel because of the contribution of the new hydrodynamic term. This is interpretable as a **vorticity** appearing.

Wilson and Cahill. Nat Commun **5**, 5075 (2014)

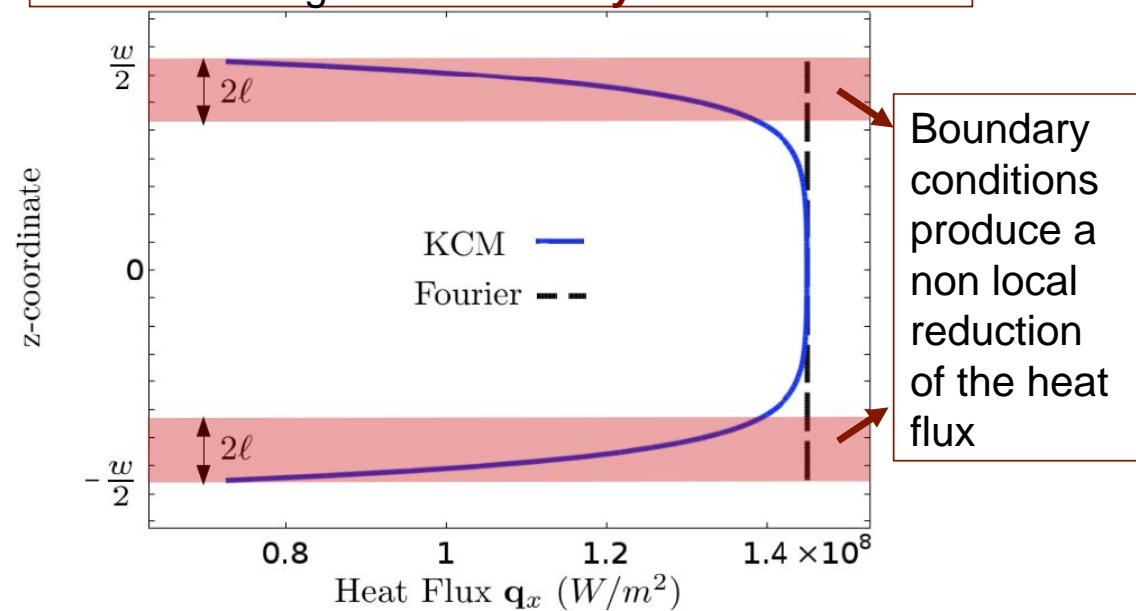


Torres et al. *Phys. Rev. Mat.*, **2**, 076001 (2018)

Silicon Thin Films

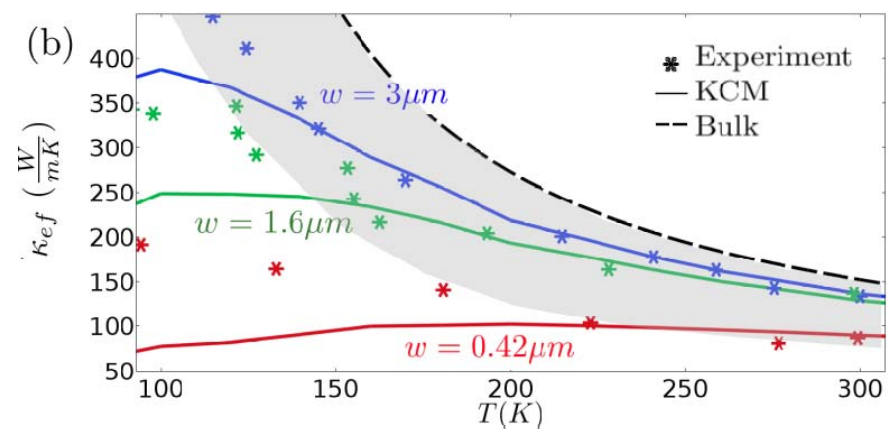
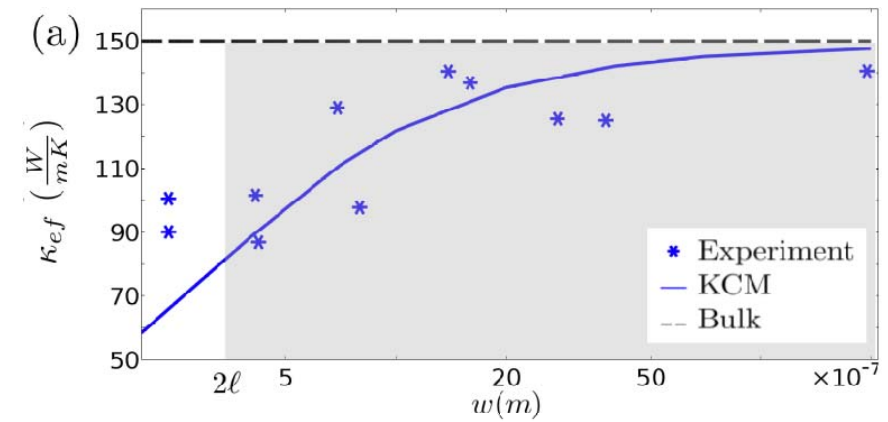
$$\kappa_{ef} = \frac{\int_{\Gamma} |\mathbf{q}| d\Gamma}{S \nabla T}$$

The effective thermal conductivity is lower than the Fourier prediction due to an effect analogous to **viscosity** in fluids.



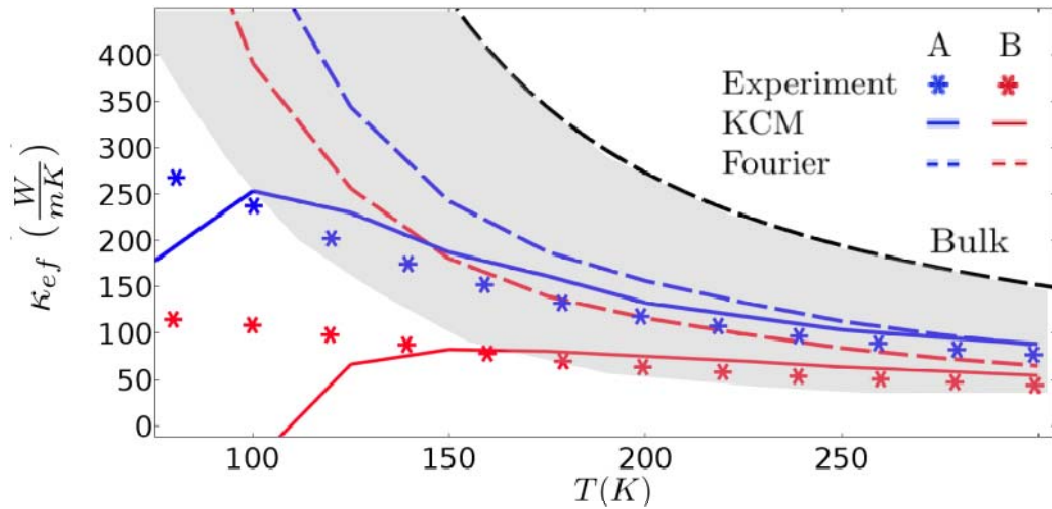
Beardo et al. submitted

Region of applicability is indicated in gray.
 $2l < w$



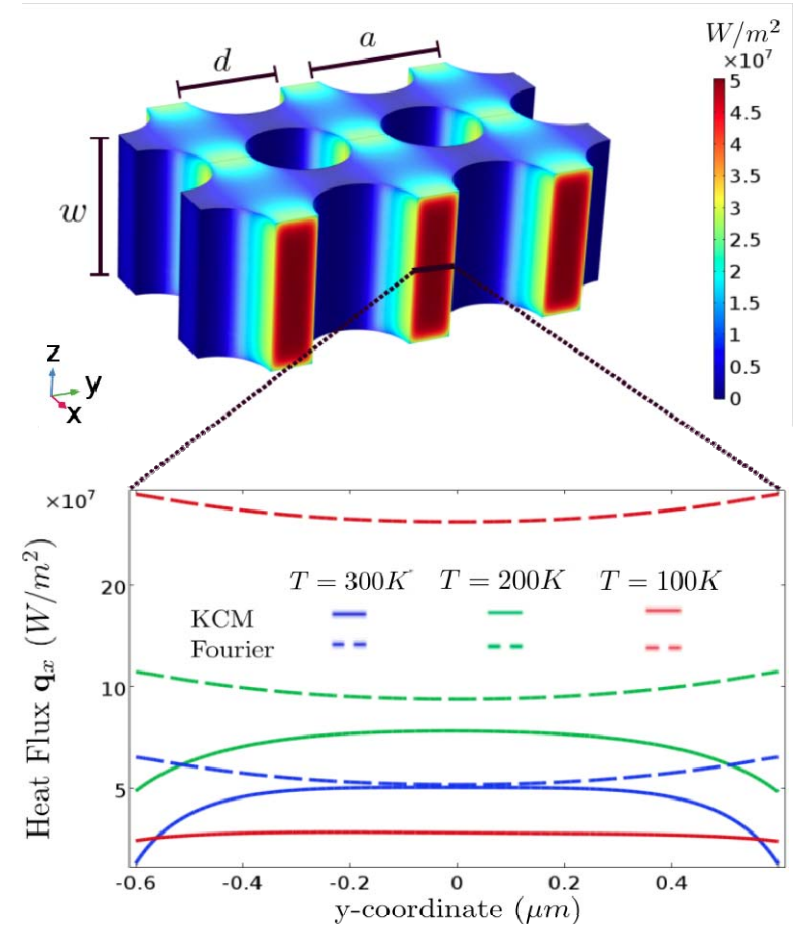
13 >> Experimental validation.

Silicon Phononic Crystals



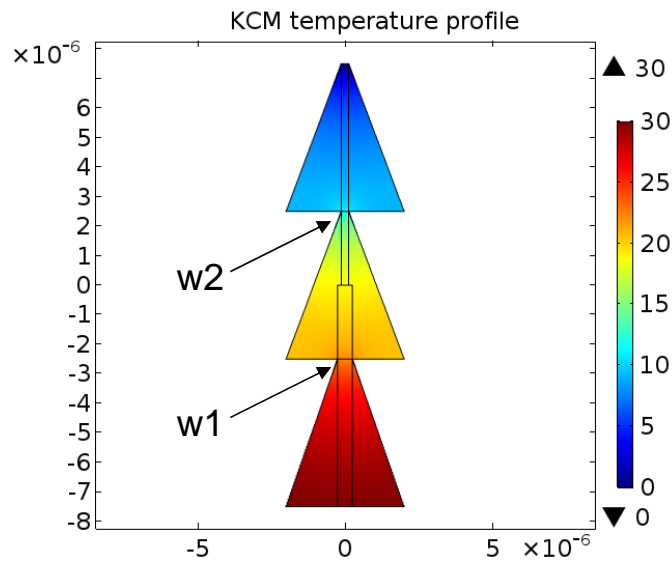
Phonic crystal A: $a = 4\mu m, d = 2.8\mu m, w = 4.49\mu m$

Phononic crystal B: $a = 20\mu m, d = 11.4\mu m, w = 4.84\mu m$

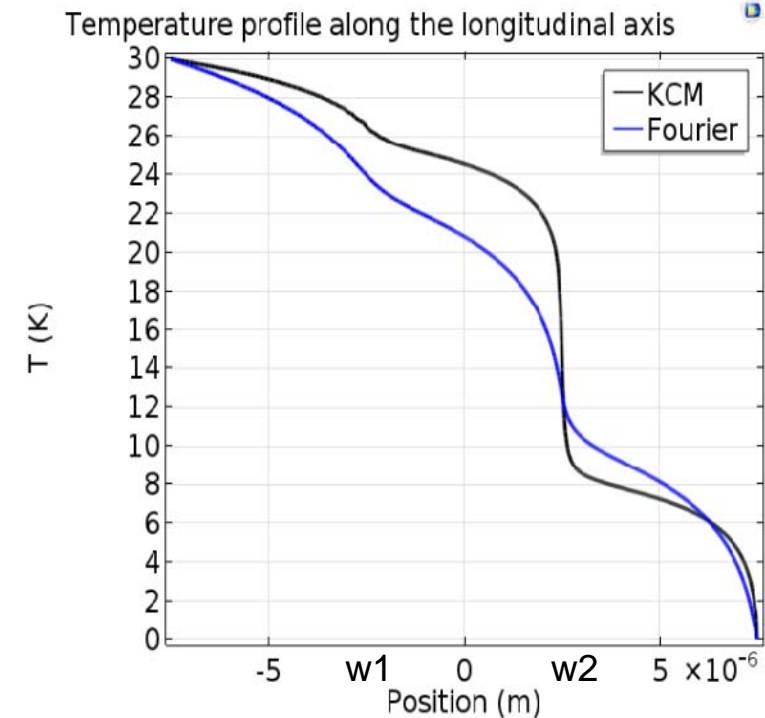


Beardo et al. submitted

Suspended Silicon Structures



Low thermal conductivity is predicted in the narrower constriction.



Experimental validation in progress by the Eindhoven University of Technology.

- Fourier law breaks down when describing thermal transport at reduced length and time scales.
- Phonon hydrodynamics is a generalization of Fourier law obtained from the microscopic description of the phonon population.
- Phonon vorticity and viscosity appear as phenomenological explanations for the thermal behavior of nano scale devices.
- Numerical implementation of the equations in a COMSOL interface allows to predict heat transport in nanoscale complex geometries.

Acknowledgements

We acknowledge the support and advice of Mats Nigam (Sweden COMSOL office) and Nicolas Huc (France COMSOL office) during the development of the Nanoscale Heat Transfer COMSOL interface.

Thanks for your attention.