Multiphysics Modeling and Multilevel Optimization of Thermoelectric Generators for Waste Heat Recovery

Asutosh Prasad, Raj CN Thiagarajan
ATOA Scientific Technologies Pvt Ltd
www.atoa.com

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Great: Develop Great Technology.

Growth:Grow into a Billion Dollar Company by 2020.

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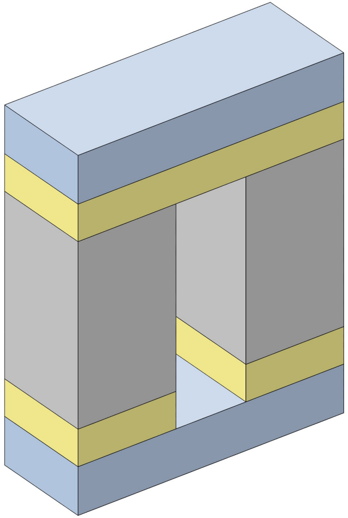
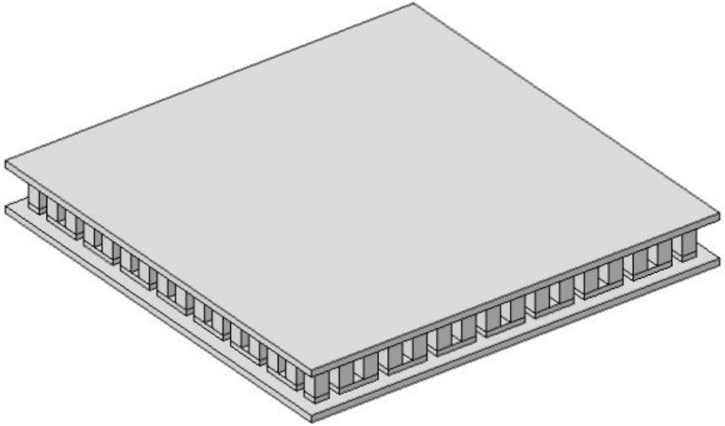
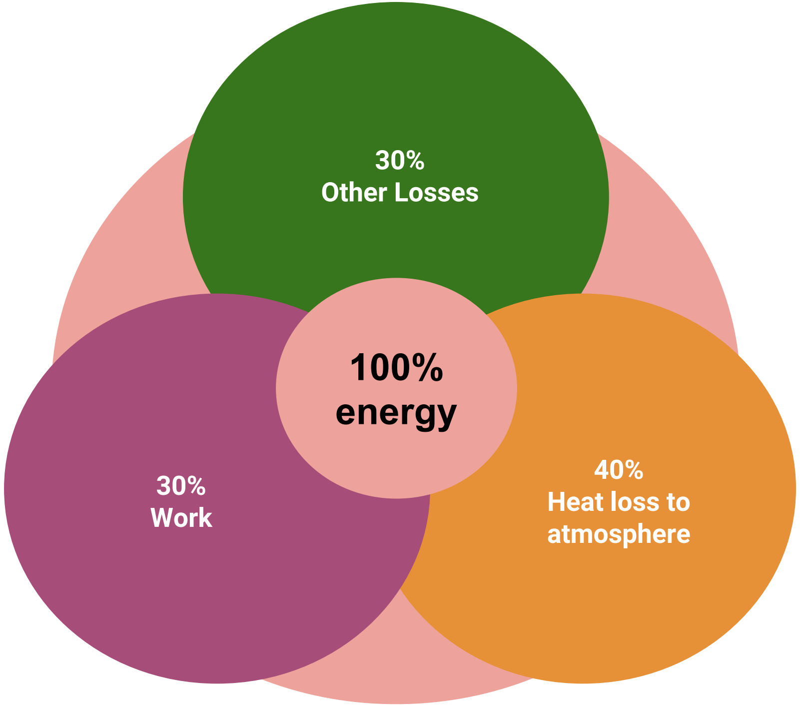




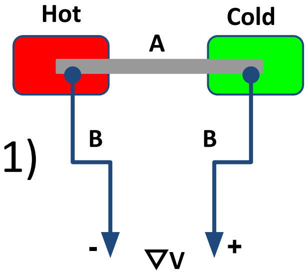
Introduction and Objective

- Waste heat is inevitable in any heat engine.
- Approximately 30% of heat energy is used for work while remaining 70% is lost to atmosphere.
- Waste heat energy reduces system efficiency and affect fuel economy.
- Thermoelectric Generation is a potential waste heat recovery technology.
- This paper explores the potential of Thermoelectric Generators for waste heat recovery.



Thermoelectric Generator (TEG)

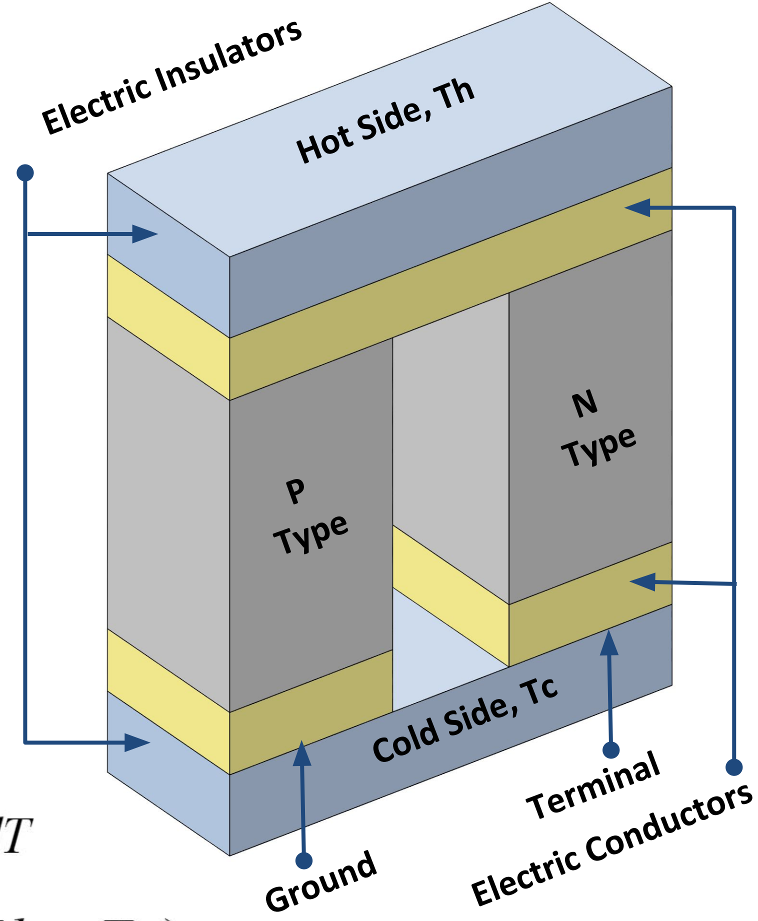
- Energy harvesting device.
- Converts temperature difference (Th-Tc) into electrical energy (Voltage).
- Thermoelectricity.
  - Seebeck effect.
  - Alessandro Volta (1794)
  - Thomas Johann Seebeck (1821)
- Thermoelectric materials
  - Bismuth Telluride (Bi_2Te_3)
  - Lead telluride (PbTe)
  - Silicon Germanium (SiGe)
  - Skutterudite ($CoAs_3$)
- Seebeck Coefficient (α)
- Thermal Conductivity (k)
- Figure of Merit (ZT)



$$\alpha = \alpha_P - \alpha_N$$

$$V_{OC} = \int_{Tc}^{Th} \alpha(T)dT$$

$$\nabla V \ or \ V_{OC} = \alpha(Th - Tc)$$



(Seebeck Effect, TEG)

Thermal System

- The thermal system of an Unit Couple TEG is represented.
- All domains are considered as thermal domains.
- As shown in the picture the thermal resistances are connected in series and parallel.
- The total thermal resistance of the unit couple is R.

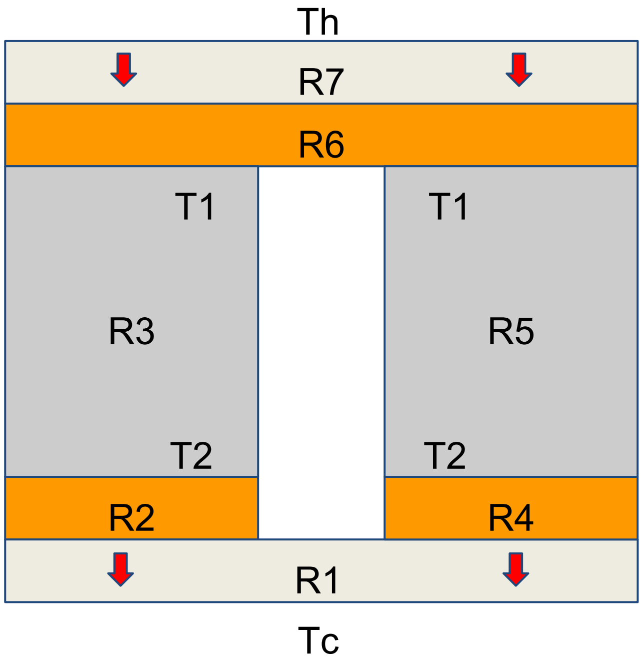
$$R = R1 + \frac{R2\,R4}{R2} + R4 + \frac{R3\,R5}{R3} + R5 + R6 + R7$$

$$Q = \frac{Th - Tc}{R}$$

$$T1 = Th - Q(R1 + R2)$$

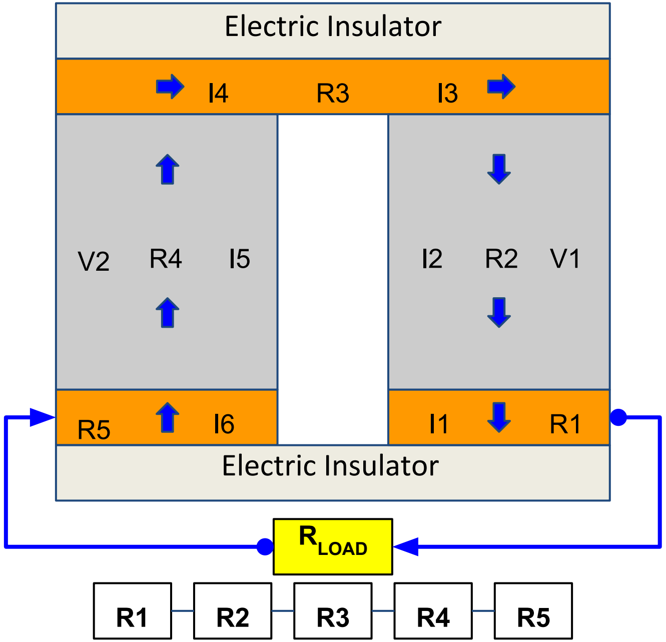
$$T2 = Th - Q(R1 + \frac{R2\,R4}{R2} + R4 + \frac{R3\,R5}{R3} + R5$$

$$\nabla T = T1 - T2$$



Electrical System

- The electrical system of unit couple TEG is represented.
- The Copper and P-Type, N-Type legs are only consider as electrical domain.
- The ceramics on the top and bottom are considered as electrical insulators.
- As shown in the picture the electrical resistances are connected in series.
- The total electrical resistance of the unit couple will be sum of all resistances.
- Total produced voltage V = V1+V2.
- Current in the system remain same since the circuit is in series.



$$R = R1 + R2 + R3 + R4 + R5 = R_{LOAD}$$

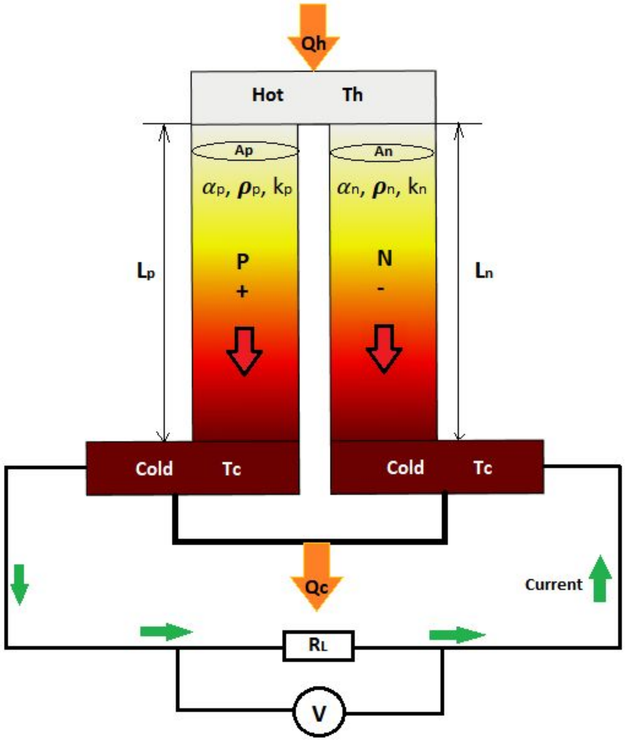
$$V = V1 + V2 = \alpha(Th - Tc)$$

$$I = I1 = I2 = I3 = I4 = I5 = I6 = \frac{V}{R + R_{LOAD}}$$

$$Power\ (P) = VI = I^2 R_{LOAD}$$

Analytical Derivation

- The analytical model for thermoelectric generator is presented.
- A schematic diagram of thermoelectric generator model is represented.
- Temperature difference is maintained at the connected junction of P-Type and N-Type thermoelectric legs.
- The produced voltage, current and power is derived analytically.



(TEG Schematic Diagram)

α = Total Seebeck Coefficient

α_p = P-leg Seebeck Coefficient

α_n = N-leg Seebeck Coefficient

ϱ_p = P-leg Electrical Resistivity

ϱ_n = N-leg Electrical Resistivity

k_p = P-leg Thermal Conductivity

k_n = N-Leg Thermal Conductivity

R = Internal Electrical Resistance

K = Total Thermal Conductance

L_p = P-Leg Length

L_n = N-Leg Length

A_p = P-Leg Cross-Sectional Area

A_n = N-Leg Cross-Sectional Area

Q_h = Heat Supplied at Hot Side

Q_c = Heat Rejected at Cold Side

R_L = Load Resistance

Analytical Derivation

Total Seebeck Coefficient

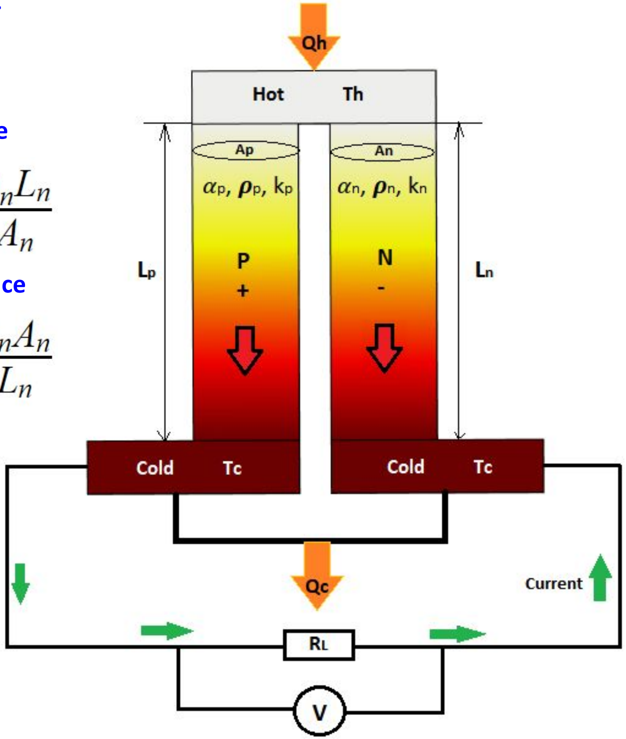
$$\alpha = \alpha_p - \alpha_n$$

Total Electrical Resistance

$$R = \frac{\rho_p L_p}{A_p} + \frac{\rho_n L_n}{A_n}$$

Total Thermal Conductance

$$K = \frac{k_p A_p}{L_p} + \frac{k_n A_n}{L_n}$$



Heat Absorbed at Hot Side

$$Q_h = \alpha T_h I - \frac{1}{2}(I^2 R) + K(T_h - T_c)$$

Heat Rejected from Cold Side

$$Q_c = \alpha T_c I - \frac{1}{2}(I^2 R) + K(T_h - T_c)$$

Power Generated

$$W = Q_h - Q_c \;, \quad W = \alpha I (T_h - T_c) - I^2 R$$

Voltage

$$V = I R_L = \alpha (T_h - T_c) - I R$$

Current

$$I = \frac{\alpha (T_h - T_c)}{R_L + R}$$

Analytical Derivation

Thermal Efficiency

$$\eta_{th} = \frac{W}{Q_h}$$

$$\eta_{th} = \frac{I^2 R_L}{\alpha T_h I - \frac{1}{2} I^2 R + K(T_h - T_c)}$$

Thermal Efficiency and Power can be written in terms of R_L / R

$$\eta_{th} = \frac{\left(1 - \frac{T_c}{T_h}\right)\left(\frac{R_L}{R}\right)}{\left(1 + \frac{R_L}{R}\right) - \frac{1}{2}\left(1 - \frac{T_c}{T_h}\right) + \frac{\left(1 + \frac{R_L}{R}\right)^2 \frac{T_c}{T_h}}{ZT_c}}$$

$$W = \frac{\alpha^2 T_c^2 \left[\left(\frac{T_c}{T_h}\right)^{-1} - 1\right]^2 \left(\frac{R_L}{R}\right)}{R\left(1 + \frac{R_L}{R}\right)^2}$$

For maximum conversion efficiency

$$\frac{d\eta_{th}}{d\left(\frac{R_L}{R}\right)} = 0 \Rightarrow \frac{R_L}{R} = \sqrt{1 + z\overline{T}}$$

Average Temperature

$$\overline{T} = \frac{T_c + T_h}{2} = \frac{1}{2} T_c \left[1 + \left(\frac{T_c}{T_h}\right)^{-1}\right]$$

Figure of Merit

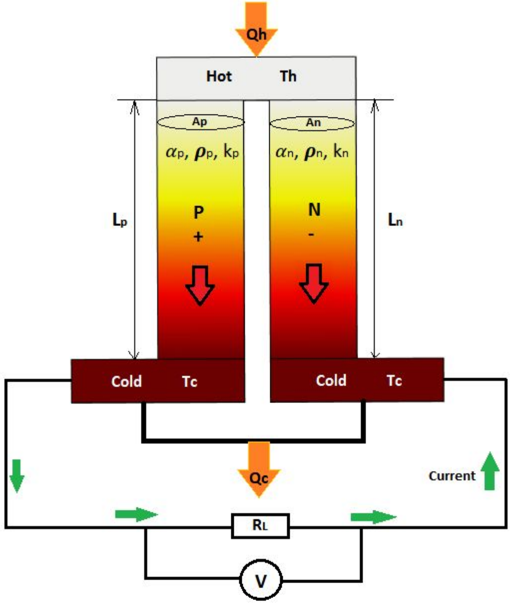
$$zT = \frac{\alpha^2 \sigma T}{k}$$

Maximum Conversion Efficiency

$$\eta_{mc} = \left(1 - \frac{T_c}{T_h} \frac{\sqrt{1 + Z\overline{T}} - 1}{\sqrt{1 + Z\overline{T}} + \frac{T_c}{T_h}}\right)$$

Analytical Derivation

- For maximum power efficiency the ratio of R_L / R is 1.
- As a result, the optimum current I_{mp} , maximum power W_{max} and maximum power efficiency η_{mp} can be derived as shown in the given equations.



For maximum power efficiency

$$\frac{dW}{d\left(\frac{R_L}{R}\right)} = 0 \Rightarrow \frac{R_L}{R} = 1$$

Optimum Current

$$I_{mp} = \frac{\alpha \nabla T}{2R}$$

Maximum Power

$$W_{max} = \frac{\alpha^2 \nabla T^2}{4R}$$

Maximum Power Efficiency

$$\eta_{mp} = \frac{\left(1-\frac{T_c}{T_h}\right)}{2-\frac{1}{2}\left(1-\frac{T_c}{T_h}\right)+\frac{4\frac{T_c}{T_h}}{ZT_c}}$$

Comsol Simulation

Governing Equations

Heat Transfer in Solid

$$\rho C_p u.\nabla T + \nabla.q = Q + Q_{ted},\ q = -k\nabla T$$

Electric Current

$$\nabla J = Q_j\ ,\ J = \sigma E + J_e\ ,\ E = -\nabla V$$

Thermoelectric Effect

$$q = PJ\ ,\ P = ST\ ,\ J_e = -\sigma S\nabla T$$

ρ = Density

u = Velocity field

C_p = Specific heat

Q = Heat source

Q_{ted} = Thermoelastic effects

Q_j = Current source

q = Heat flux in conduction

K = Thermal conductivity

T = Temperature

P = Peltier Coefficient

J = Induced Electric Current

J_e = External Current Source

E = Electric field

V = Electric Potential

S = Seebeck Coefficient

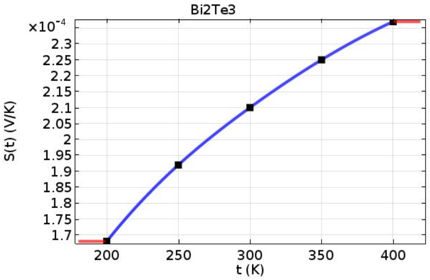
σ = Electrical Conductivity

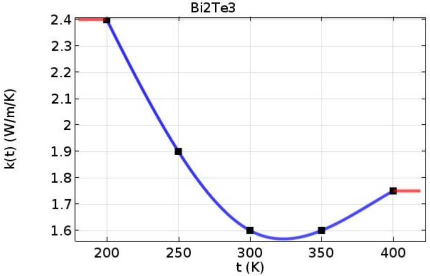
Material Properties

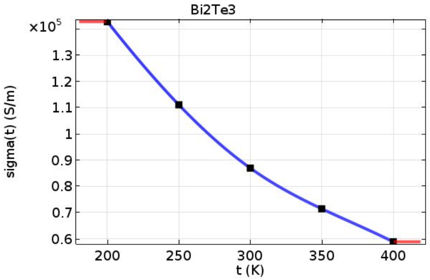
- The thermoelectric material properties such as Seebeck Coefficient, Electrical Conductivity, Thermal Conductivity are given in the below table.
- The temperature dependent material properties are represented graphically.
- The Seebeck Coefficient of Bi2Te3 is positive for P-Type while negative for N-Type.
- Thermal conductivity and electrical conductivity of Bi2Te3 is decreases with increase in temperature.

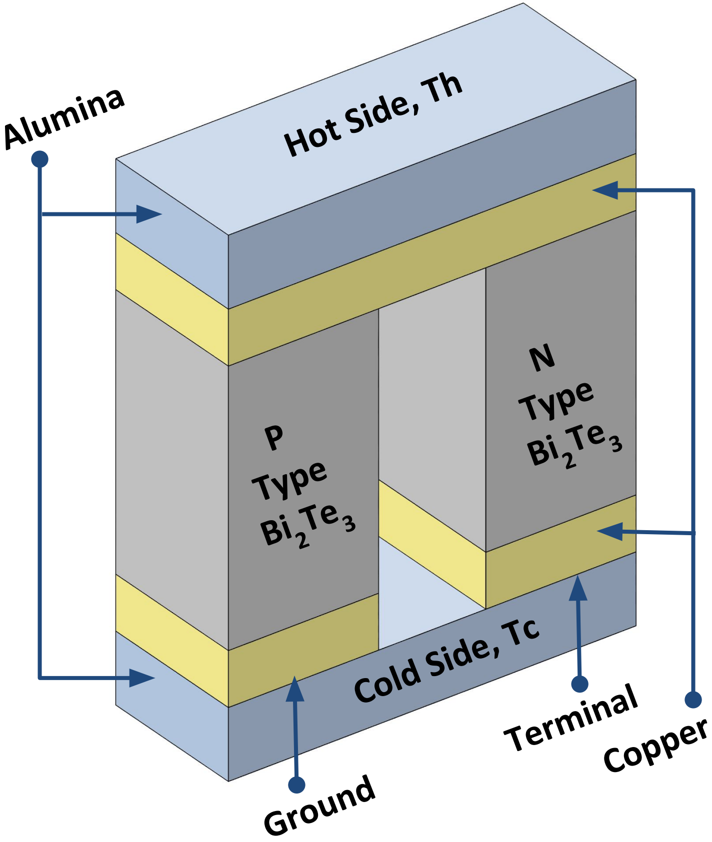
Materials	ρ (kg/m3)	k (W/mK)	σ (S/m)	Cp (J/kg.K)	S (V/K)
Bi2Te3(P)	7700	k(T)	sigma(T)	154	S(T)
Bi2Te3(N)	7700	k(T)	sigma(T)	154	-S(T)
Copper	8700	400	5.998E7	385	0.0
Alumina	3900	27	0.0	900	0.0

$$zT = \frac{\alpha^2 \sigma T}{k} = 1$$





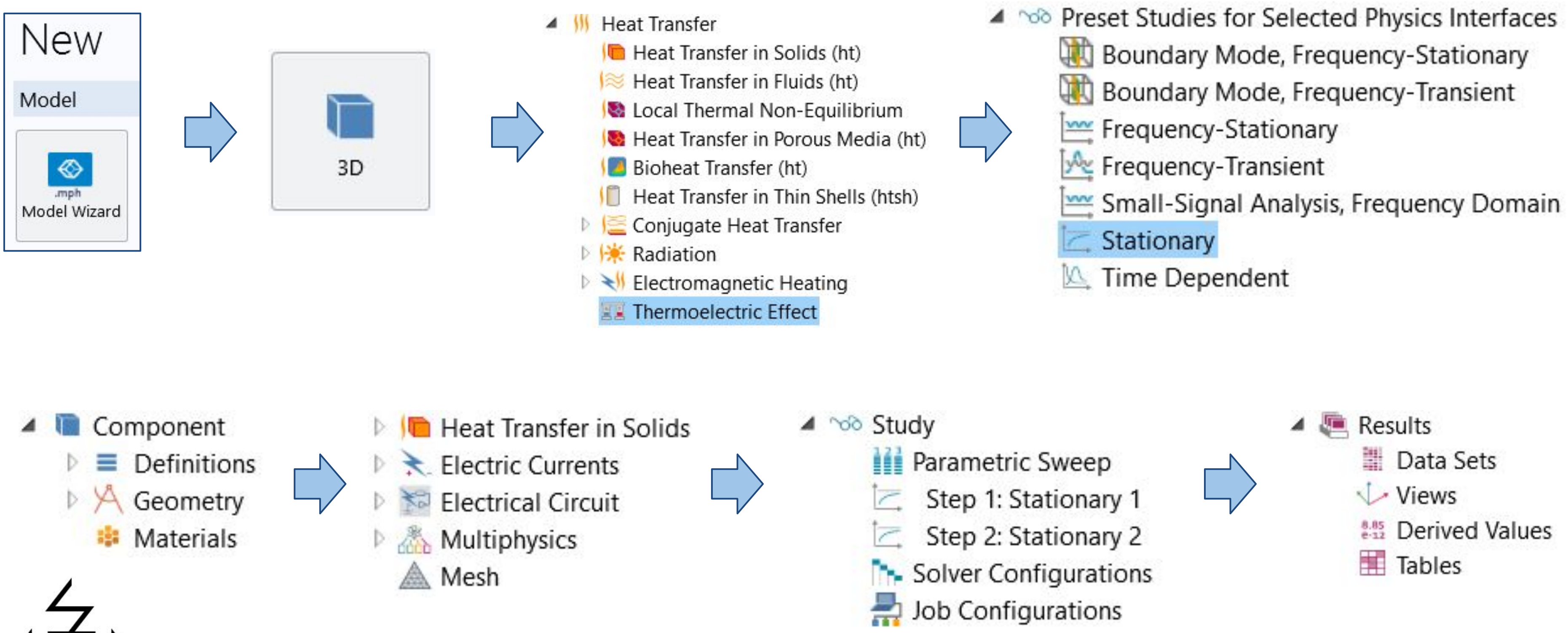




(Material Definition)

Simulation in Comsol

Steps



Unit Couple TEG

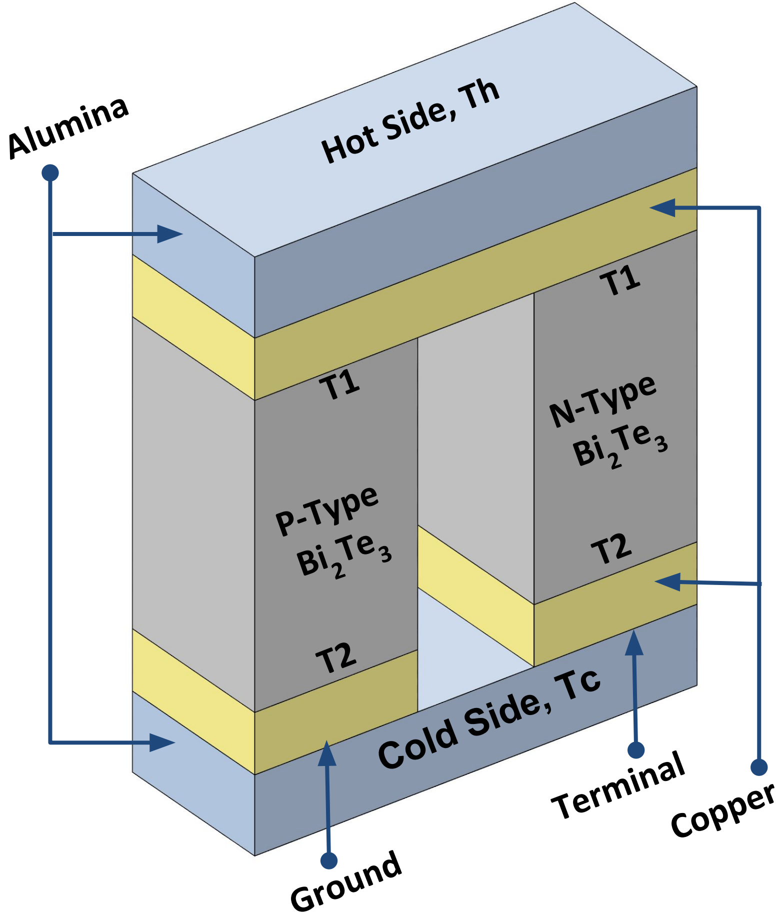
Voltage (∇*V*), Current (I) and Power (W) from Temperature gradient (∇*T*)

Effects of Temperature gradient (**∇T**) on Voltage (**∇*V***), Current (***I***) and Power (***W***) is calculated for the Unit Couple TEG, thermally in parallel and electrically in series.

P/N Leg Width = 1.4 mm
P/N Leg Depth = 1.4 mm
P/N Leg Height = 2.5 mm
P/N Leg cross-sectional area = 1.96 mm^2
Unit Couple Total Height = 4.8 mm

Pitch = 1 mm
Copper Thickness = 0.5 mm
Alumina Thickness = 0.65 mm

Load resistance = 0 to 1Ω
Cold side temperature Tc = 20^C
Hot side temperature Th = 100^C to 300^C

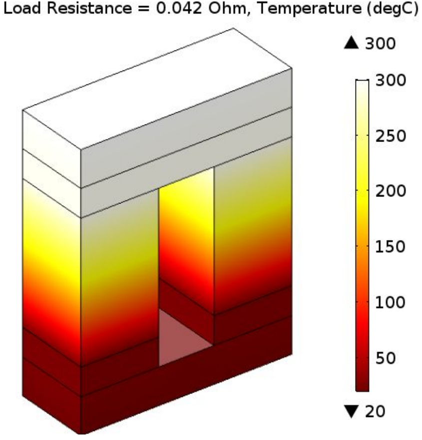


(Unit Couple TEG)

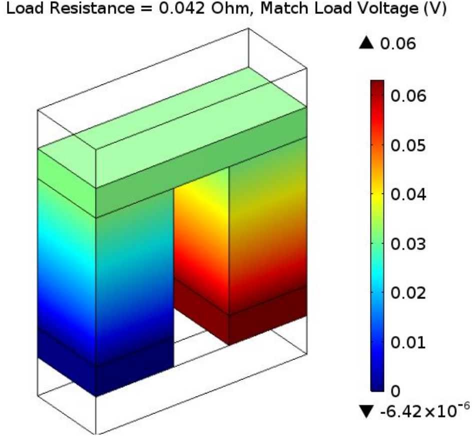
Unit Couple TEG Simulation Results:-

Close Circuit, Contour Plots:-

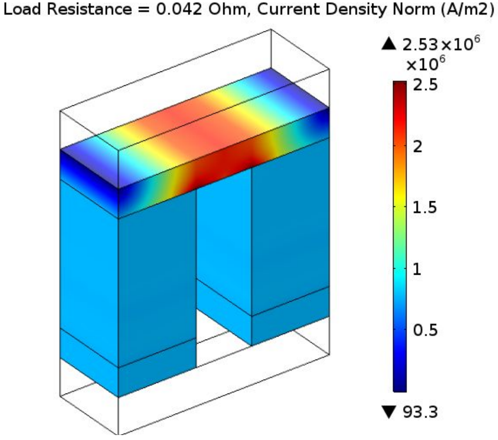
- A stationary study is assigned for the computational model.
- A fully coupled direct solver is implemented.
- The Temperature (C), Voltage (V), Current Density(A/m^2) and Seebeck Coefficient (V/K) of Unit Couple TEG are plotted respectively at maximum temperature difference.
- The maximum temperature **300^C** and minimum of **20^C**.
- The match load voltage or optimal voltage obtained is **0.06V** when internal resistance and load resistance are equal at **0.042Ω**.



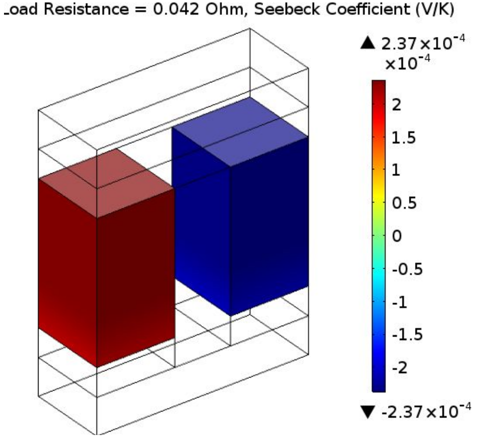
Temperature



Match Load Voltage, Vopt



Match Load Current Density

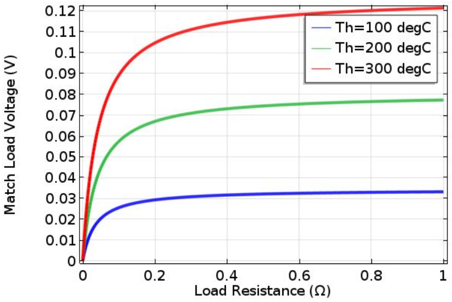


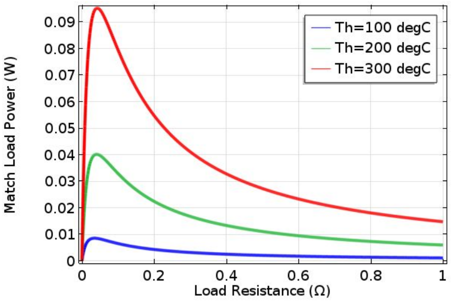
Seebeck Coefficient

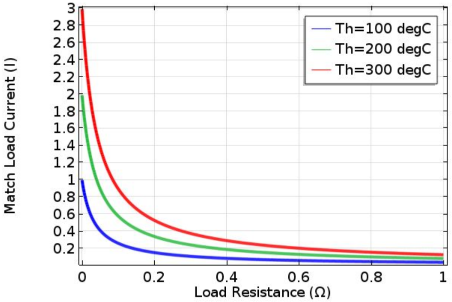
Unit Couple TEG Simulation Results:-

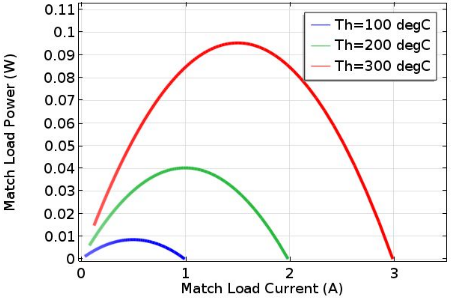
Close Circuit, Graphical Plots:-

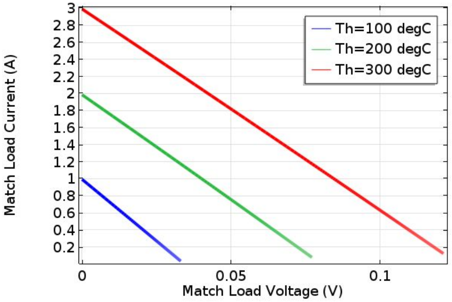
- The derived values from Unit Couple TEG simulation are plotted graphically.
- A Maximum Power of **0.095W** is produced from the Unit Couple TEG at **0.042Ω** and **280^{C}** temperature difference.
- The Optimum Voltage, Current and Power of **0.06V * 1.59A = 0.095W** is produced from Unit Couple TEG at **0.042Ω** and **280^{C}** temperature difference.
- A Maximum Voltage of **0.12V** is produced from Unit Couple TEG at **1Ω and 280^{C}** temperature difference while the power is **0.0147W.**
- Maximum efficiency of **10%** is achieved from the Unit Couple TEG .

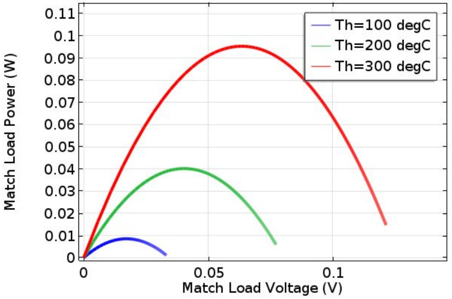












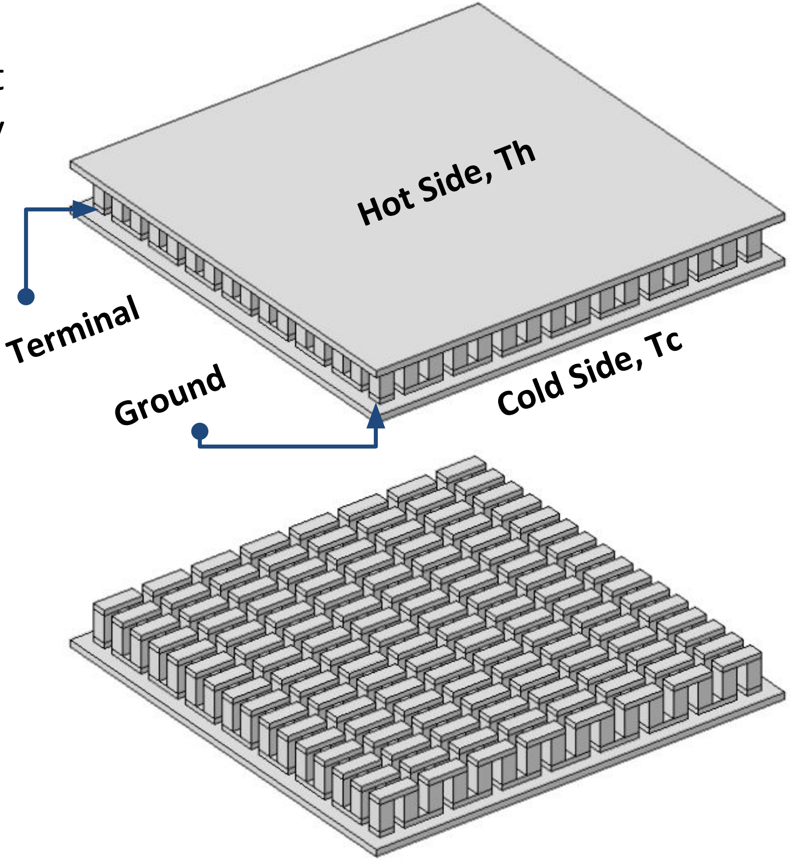
TEG Module

Voltage (∇*V*), Current (I) and Power (W) from Temperature gradient (∇*T*)

Effects of Temperature gradient (**∇T**) on Voltage (**∇*V***), Current (***I***) and Power (***W***) is calculated for the TEG Module, thermally in parallel and electrically in series.

P/N Leg Width = 1.4 mm
P/N Leg Depth = 1.4 mm
P/N Leg Height = 2.5 mm
P/N Leg cross-sectional area = 1.96 mm^2
Unit Couple Total Height = 4.8 mm
Number of Thermocouple = 128
Pitch = 1 mm
Copper Thickness = 0.5 mm
Alumina Thickness = 0.65 mm

Load resistance = 0 to 80Ω
Cold side temperature Tc = 20^C
Hot side temperature Th = 100^C to 300^C

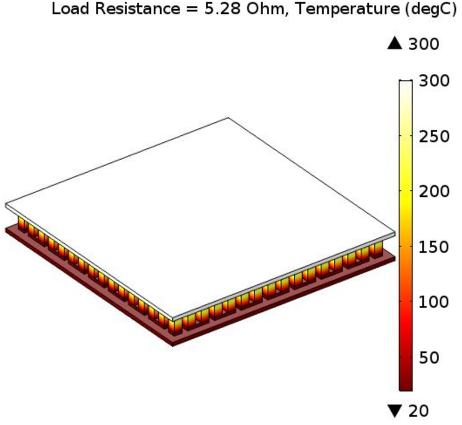


(TEG Module)

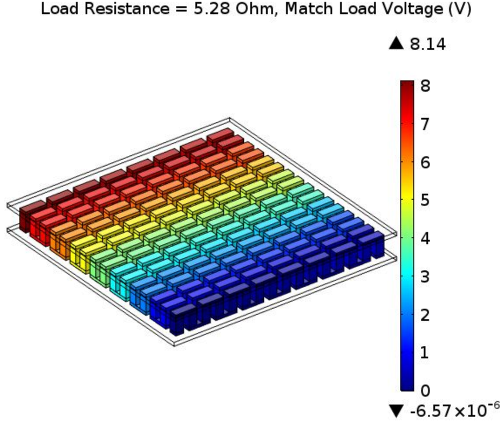
TEG Module Simulation Results:-

Close Circuit, Contour Plots:-

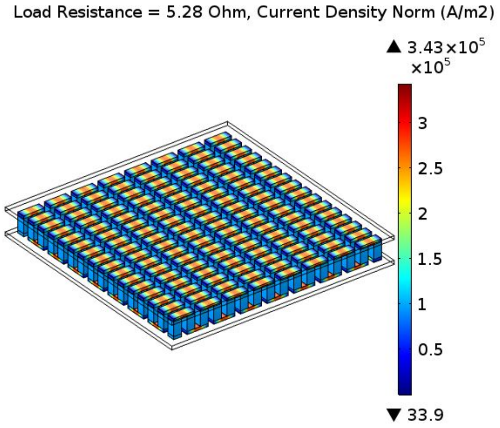
- A stationary study is assigned for the computational model.
- A fully coupled direct solver is implemented.
- The Temperature(C), Voltage (V), Current Density(A/m^2) and Seebeck Coefficient (V/K) of Unit Couple TEG are plotted respectively at maximum temperature difference.
- The maximum temperature **300^C** and minimum of **20^C**.
- The match load voltage or optimal voltage obtained is **8.14V** when internal resistance and load resistance are equal at **5.28Ω**.



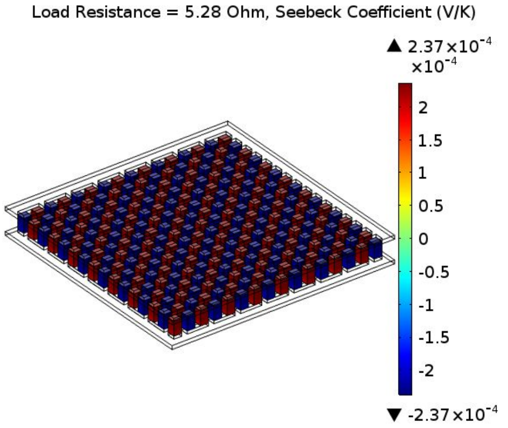
Temperature



Match Load Voltage, Vopt



Match Load Current Density

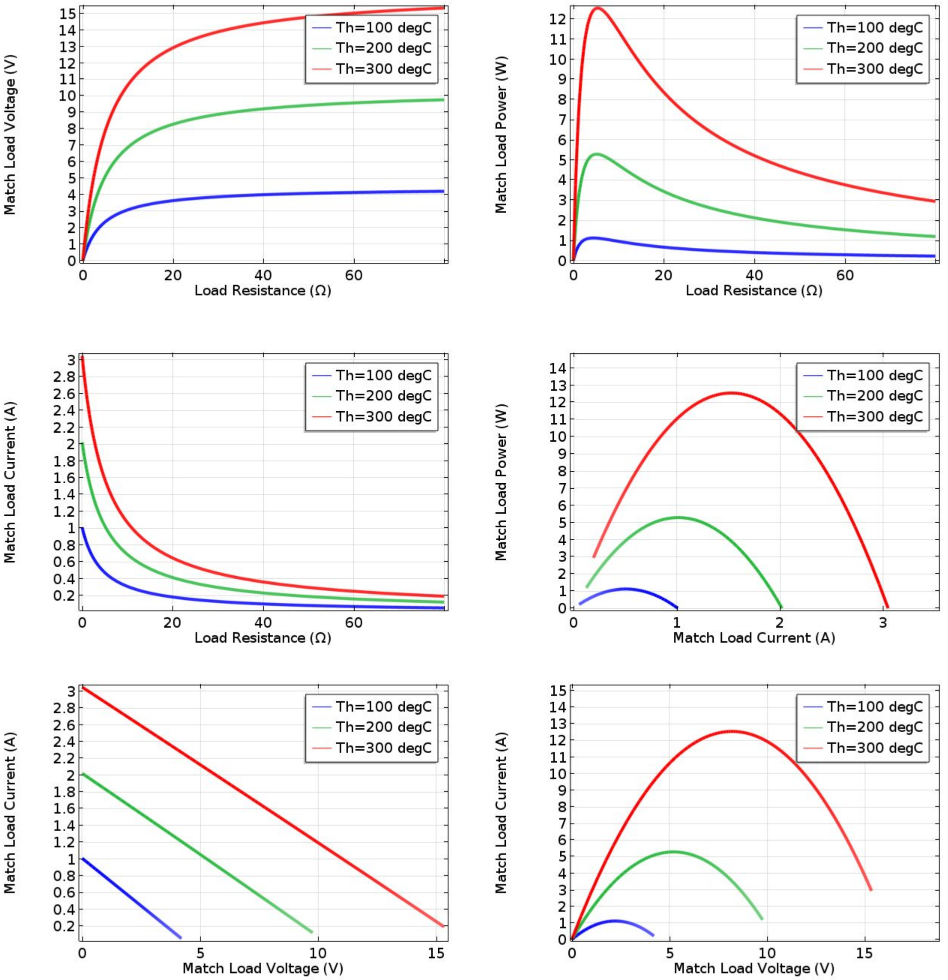


Seebeck Coefficient

TEG Module Simulation Results:-

Close Circuit, Graphical Plots:-

- The derived values from TEG Module simulation are plotted graphically.
- A Maximum Power of **12.59W** is produced from the TEG Module at **5.28Ω** and **280^{C}** temperature difference.
- The Optimum Voltage, Current and Power of **8.14V * 1.5A = 12.59W** is produced from TEG Module at **5.28Ω** and **280^{C}** temperature difference.
- A Maximum Voltage of **15.33V** is produced from TEG Module at **80Ω** and **280^{C}** temperature difference while the Power is **2.94W.**
- Maximum efficiency of **10%** is achieved from the TEG Module.



Conclusion and Future Work

- In this paper the working principle and simulation of thermoelectric generator are explained both theoretically and numerically.
- An Unit Couple TEG and Optimized Multilevel TEG Module consisting arrays of thermocouples is designed and simulated in COMSOL
- The performance of Unit Couple TEG and TEG Module are investigated numerically.
- The optimized multilevel modeling of thermoelectric generators shows potential in maximum conversion of waste heat into useful electric power.

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