**INTRODUCTION**: This paper conducted incompressible flow computations based on the variational multiscale method and reports some numerical results of various incompressible flow fields, including flow past a circular cylinder immersed in a channel between parallel plates. Finite element analysis of the incompressible Navier-Stokes equations based on a residual-based

**RESULTS**: The results for the Incompressible unsteady flow around a circular cylinder of Re=3900 are displayed in Figs.1 and 3. The computational condition is shown in Fig.2. It is shown that the present result is consistent with experiment after the correction of the blockage effect.

Measurement  $C_d = 0.98 \pm 0.05$ 

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Based on a Variational Multiscale Method

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## Finite-Element Analysis of Unsteady Flow Past a Circular Cylinder

variational method was performed. In the present actual computation, the residual-based variational multiscale (RBVM) model<sup>1)</sup> which has been implemented in the latest version of CFD module of COMSOL Multiphysics<sup>®</sup> Ver.5.4 was utilized here.



Figure 1. Snapshots of unsteady flow around a circular cylinder

**COMPUTATIONAL METHODS**: The Navier-Stokes equations with the continuity equation are solved.



**Figure 3**. Maskell's formula <sup>7)</sup> and the resulting corrected value of the coefficient of drag of circular cylinder.

**Applicability of RBVM**: Various flow fields were also simulated by using RBVM.



**Figure 4**. Hele-Shaw flow of Re=2 and comparison with van Dyke <sup>4</sup>).





The variational multiscale method has been proposed by Hughes (1995)<sup>2)</sup>. His formula is expressed in terms of the classical Green's function and the projector which defines the decomposition of the solution into coarse and fine scales. Bazilevs et al.<sup>3)</sup> developed an LES-type variational theory of turbulence, where any ad hoc devices, such as eddy viscosities, are not employed. In order to compute the fine-scale field, the element-wise stabilization operator  $\tau$  is computed from the formula for the fine-scale Green's operator, and is represented as the product of  $\tau$  and the local coarse-scale residual. This formulation has a remarkable character: if the coarse-scale (i.e., grid scale) is fully resolved, the fine-scale model becomes zero. This means this



**Figure 5**. Flow past a circular cylinder of Re=40 and comparison with Kawaguchi <sup>5)</sup>.



**Figure 7**. Flow past a semi-sphere on the ground of Re=40000, resembling the horseshoe vortex system of Fedrizzi et al.<sup>8)</sup>

**CONCLUSIONS**: Based on the present study, it is expected that the RBVM of CFD module of COMSOL Multiphysics<sup>®</sup>Ver.5.4 will discover important features of fluid dynamics, in the wide range of the flow Reynolds number, including laminar, transition and turbulent flow.

### model is consistent.

# All of computations here is executed based on the three-dimensional and time-dependent study.

Plate



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