Three-Dimensional Mixed Convection in a Rectangular Duct

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INTRODUCTION

Numerical study is performed for air flow in a horizontal and inclined (15° & 30° to the x-y plane) three-dimensional duct in which one side is heated and three others insulated. Depending on the flow parameters, the dominant mode of heat transfer is mixed convection. The continuity, momentum, and energy equations of the mixed convection heat transfer are solved using a numerical (FEA) approach. All simulations were conducted with COMSOL Multiphysics simulation software and the application modules utilized are the CFD module and the heat transfer module. This work focuses on parameter ranges of $100 \le Re \le 1000$ and $-8 \le Ri \le 800$, where $Ri = Gr/Re^2$. The effects of the changes in ***Re*** and ***Ri*** on the air flow in the duct are investigated.

NUMERICAL METHOD VALIDATION

A numerical study of air flow and heat transfer in a finite, vertical duct (Figs. 1 & 2) of aspect ratio (=height/width), **A = 12,** with the same ranges of ***Re*** and ***Gr*** was investigated using COMSOL Multiphysics to validate results obtained from the software. Results obtained show good agreements with the published paper by Yang et. al. [1]. A superposed plot, Fig. 3, of local Nusselt number along the heated surface shows a maximum difference of 15.38% between the published work and the present work for ***Re*** = 200 and ***Ri*** =-3.

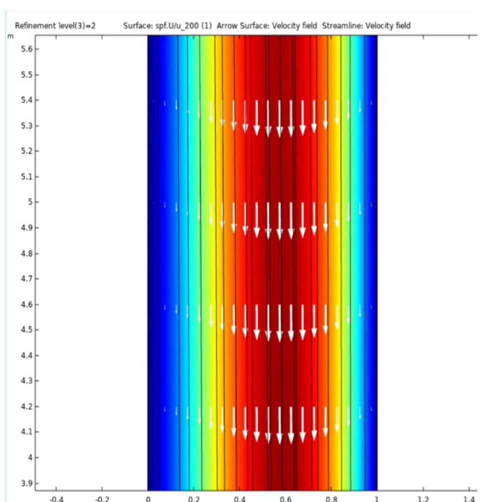

Fig. 1 Velocity plot for ***Re*** = 200, ***Ri*** = -3

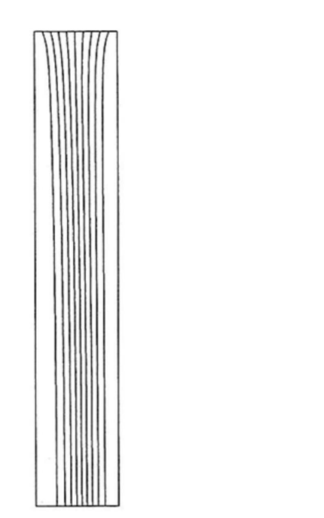

Fig. 2 Streamline plot for ***Re*** = 100, ***Ri*** = -3 by Yang et. al. [1]

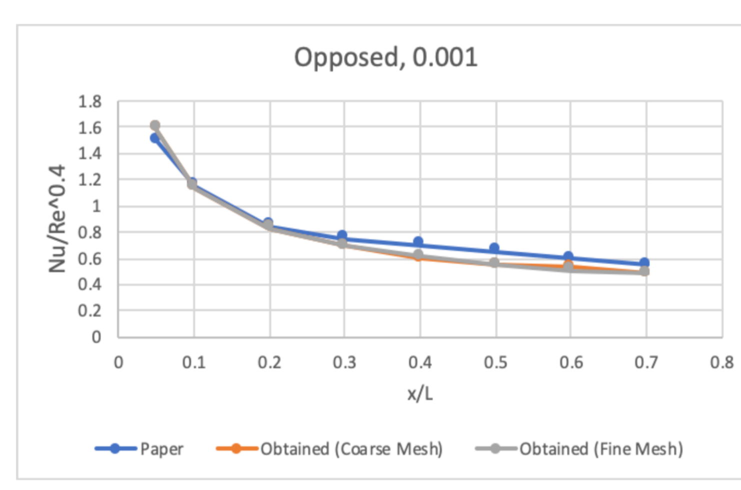

Fig. 3 Superposed plot of local Nusselt number from both published work and generated result for ***Re*** = 100, ***Ri*** = -3

PROBLEM DEFINITION

Flow and heat transfer characteristics are obtained for horizontal and inclined rectangular ducts with dimensionless geometric ratios of L:W:H = 12:1:1. The inclined ducts have inclination angles of 15° & 30° to the x-y plane. Opposed flow (in which flow enters the duct from the top open surface) and assisted flow (in which flow enters the duct from the bottom open surface) are considered in this work. The left wall is heated uniformly and the right, top, and bottom walls are perfectly insulated (Fig. 4). Steady-state simulations are carried out for the selected parameter ranges of ***Re*** and ***Ri***. All appropriate properties of the fluid are determined at room temperature and atmospheric pressure.

Boundary Conditions

Y = 1: $\mathbf{u} = 0$, $\partial\theta/\partial X = -1$, $\partial\theta/\partial Z = -1$; Y=0, Z=0, Z=1: $\mathbf{u} = 0$, $\partial\theta/\partial Y = 0$

X=0: u = 1, v = 0, w = 0, $\theta = 0$; X=12: P = 1 atm

Where $\theta = (T - T_o)/(qD_h/k)$, k = thermal conductivity, D_h = hydraulic diameter, q = heat flux

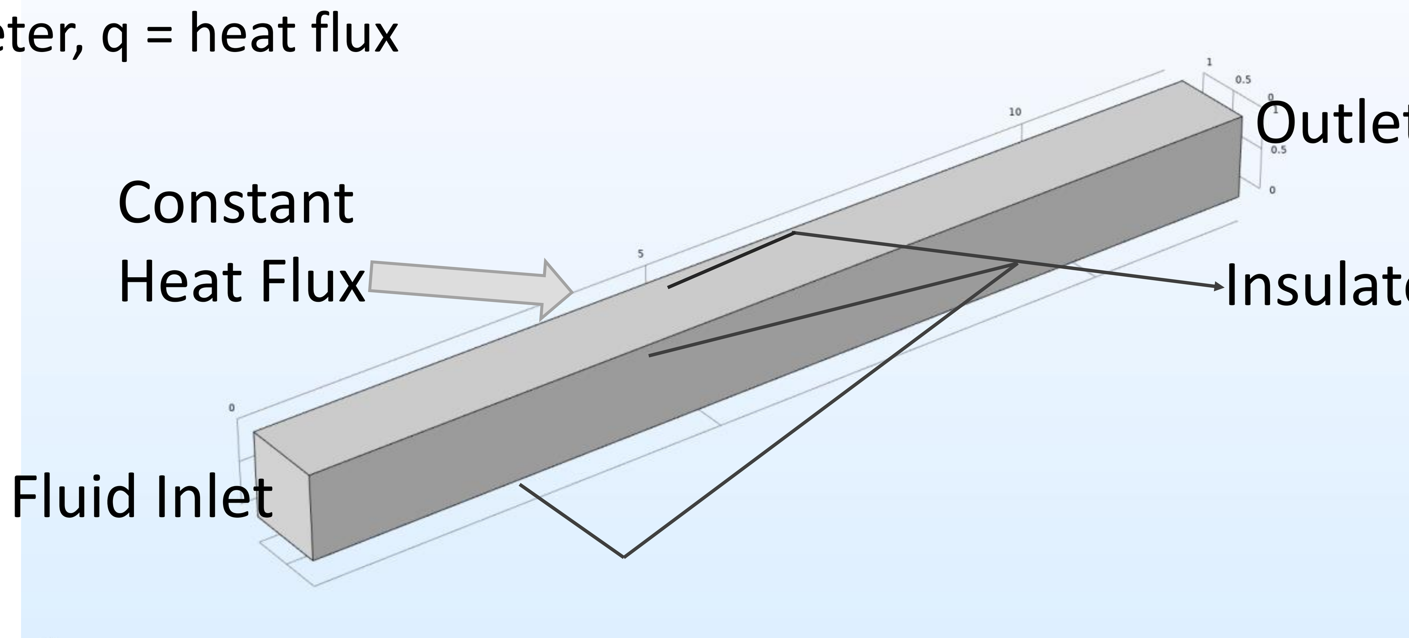

Fig. 4 Boundary Layout

IMPLEMENTATION OF SOLUTION METHOD

Incompressible flow is assumed with the Boussinesq approximation accounting for the buoyancy variation, and reduced pressure simplifying the density term in the momentum equation. Using the default relative tolerance of **0.001**, the maximum number of iterations and mesh size are varied for the flow especially ones which converge slowly. In most cases, a maximum iteration number of **300** and extra coarse mesh size were the best compromise between precision and computation time. The pertinent governing equations of COMSOL Multiphysics are as follows:

$$\rho(\mathbf{u}\cdot\nabla)\mathbf{u} = \nabla\cdot[-p\mathbf{I}+\mathbf{K}] + \mathbf{F} + (\rho - \rho_{ref})\mathbf{g}$$

$$\rho\nabla\cdot(\mathbf{u}) = 0$$

$$\mathbf{K} = \mu\left(\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right)$$

$$\rho C_p\mathbf{u}\cdot\nabla T + \nabla\cdot\mathbf{q} = Q + Q_p + Q_{vd}$$

$$\mathbf{q} = -k\nabla T$$

These equations, in addition to the boundary conditions specified earlier, allow solutions to be obtained for the fluid flow and heat transfer characteristics in the specified geometry.

RESULTS

- For a selected plane, the increase in inclination angle from 0° to 15° and 30° shows no significant difference in the streamline plot for the range of parameters in this study (Figs. 6, 7 & 12).
- For horizontal, assisted, and opposed flow, a reverse flow is initiated in the downstream and an increase in ***Re*** pushes the reverse flow downstream (Figs. 8 & 11, 13 & 16); however an increase in ***Ri*** pushes the reverse flow inwards (Figs. 5 & 6, 11 & 12, 15 & 16)
- Huge difference between ***Re*** and ***Ri*** values result in larger vortices in the duct (Figs. 9 & 14)

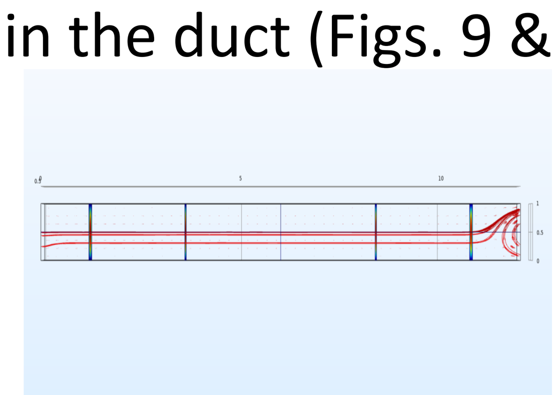

Fig. 5 ***Re*** = 200 ***Ri*** = 100 Horiz.

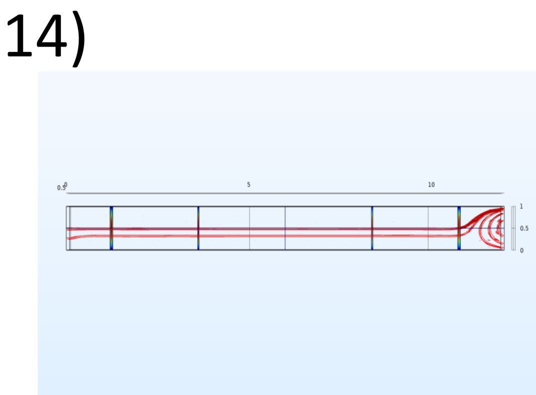

Fig. 6 ***Re*** = 200 ***Ri*** = 400 Horiz.

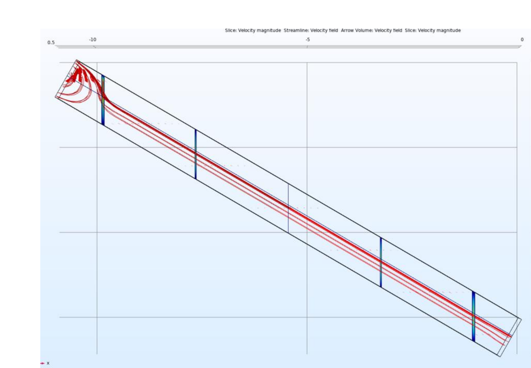

Fig. 7 ***Re*** = 200 ***Ri*** = 400 Ass. 30°

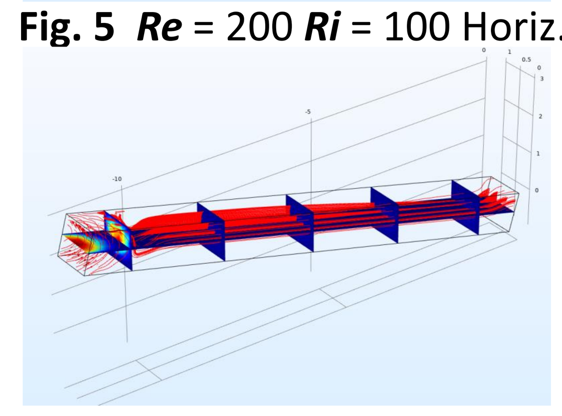

Fig. 8 ***Re*** = 100 ***Ri*** = 100 Ass. 15°

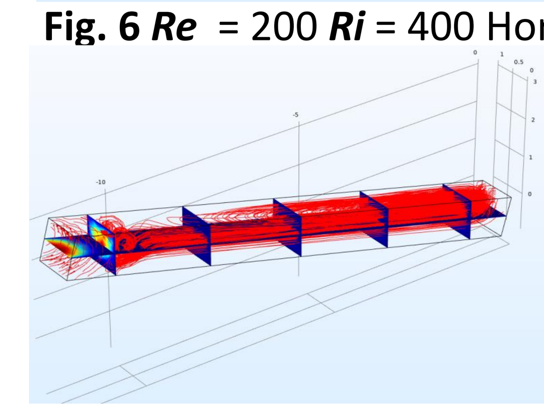

Fig. 9 ***Re*** = *100* ***Ri*** = *400* Ass. 15°

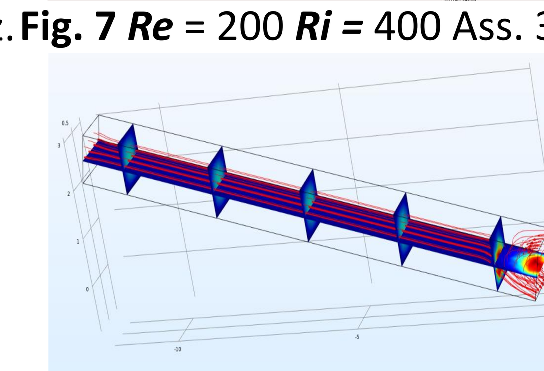

Fig. 10 ***Re*** = 100 ***Ri*** = -2 Opp. 15°

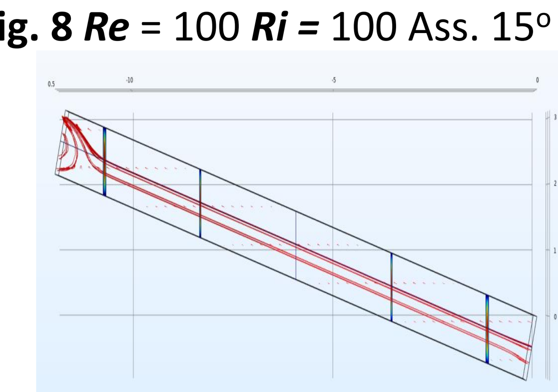

Fig. 11 ***Re*** = 200 ***Ri*** = 100 Ass. 15°

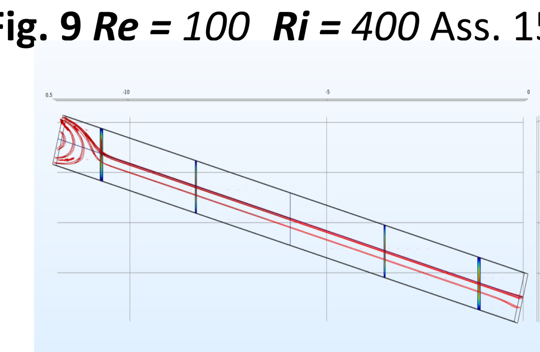

Fig. 12 ***Re*** = 200 ***Ri*** = 400 Ass. 15°

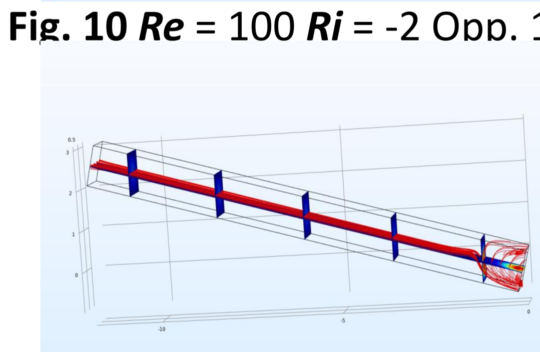

Fig. 13 ***Re*** = 100 ***Ri*** = -8 Opp. 15°

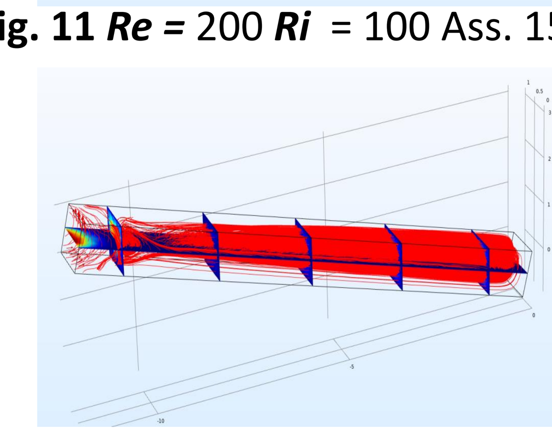

Fig. 14 ***Re*** = 1000 ***Ri*** = 100 Ass. 15°

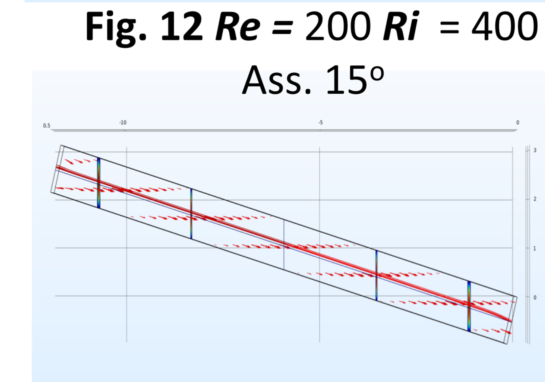

Fig. 15 ***Re*** = 200 ***Ri*** = -3 Opp. 15°

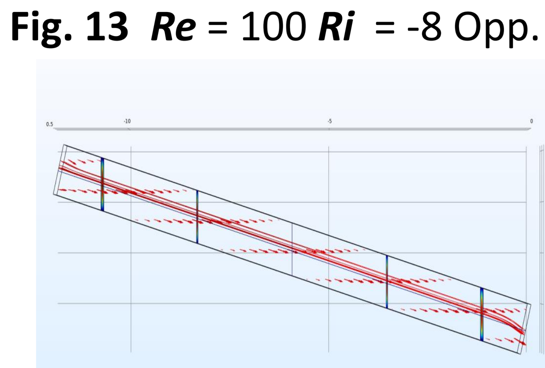

Fig. 16 ***Re*** = *200* ***Ri*** = neg 8 Opp. 15°

CONCLUSIONS AND FUTURE WORK

Limited results for mixed convection in horizontal and inclined rectangular duct have been presented in this work. Results compare very well for published work on two-dimensional vertical flow channel. Future work will extend this study to further validate the numerical procedure with the results obtained by Incropera and Schutt [2]. Ultimately, the intent of this research is to obtain flow and heat transfer characteristics for the design of novel heat exchangers such as solar collectors.

SELECTED REFERENCES:

1. Yang, C. S., D. Z. Jeng, K. A. Yih, C. Gau, and Win Aung. "Numerical and Analytical Study of Reversed Flow and Heat Transfer in a Heated Vertical Duct." *Journal of Heat Transfer* 131, no. 7 (2009): 072501. doi:10.1115/1.3109998.
2. Incropera, F. P., and J. A. Schutt. "Numerical Simulation of Laminar Mixed Convection in the Entrance Region of Horizontal Rectangular Ducts." Numerical Heat Transfer 8, no. 6 (1985): 707-729.

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