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Validation of a Simplified Model to Determine the Long-Term Performance of Borehole Heat Exchanger Fields with Groundwater Advection

S. Lazzari [1], A. Priarone [2] and E. Zanchini [1]

[1] DIENCA, Università di Bologna, [2] DIPTEM-TEC, Università di Genova
*Corresponding author: Stefano Lazzari, DIENCA, Viale Risorgimento 2, I-40136, Bologna, Italy, stefano.lazzari@unibo.it

Abstract: Finite element simulations performed through COMSOL Multiphysics (© COMSOL AB.) are used to study the long-term performance of BHE fields placed in a water-saturated porous soil subjected to groundwater movement. The heat transfer problem is written in a dimensionless form and the long-term time evolution of the mean surface temperature of the BHEs, sketched as cylindrical heat sources subjected to a regular time periodic heat load, is determined with reference to a single line of three BHEs and to a square field of twenty-five BHEs.
A simplified model to determine the long-term performance of BHE fields, based on the superposition principle, is proposed and validated. The accuracy of the simplified model is found to be excellent for design purposes.

Keywords: Borehole Heat Exchanger (BHE) Fields, Groundwater Movement, Long-Term Operation, Superposition Principle, Finite Element Simulations.

1. Introduction

Borehole Heat Exchangers (BHEs) are used to exploit the so-called "low enthalpy geothermal resources", *i.e.* energy resources which are available at temperatures that are not suitable for the production of electricity or direct heating of buildings, but allow an advantageous use of heat pumps for space heating and cooling. In the most widely used arrangement, BHEs are composed of two U-bent polyethylene vertical tubes that are placed in a borehole which is then grouted. The fluid which flows inside the tubes, typically a water-glycol solution, is responsible for the heat transfer with the ground.

The total length of the BHE field necessary for a plant is usually determined by means of the design method recommended by ASHRAE [1], which is based on the analytical solution proposed by Carlslaw and Jaeger [2] for the temperature field around a cylindrical surface placed in an infinite solid medium and subjected to a constant heat flux per unit area. The ASHRAE method considers the superposition of three constant heat pulses, with a duration of one year, one month and six hours, respectively.

In the case of unbalanced winter and summer thermal loads and negligible groundwater movement, either an insufficient distance between BHEs or a too high thermal load per unit length can lead to the system collapse in a few decades, even if it is designed according to the ASHRAE method, which is based on a simplified analysis over a period of 10 years.

Thus, the study of the long-term efficiency of BHE fields has received a considerable attention in recent literature.

The sustainability of a single BHE with completely unbalanced winter and summer loads has been assessed both theoretically and experimentally, even in the case of absence of groundwater flow [3, 4]. On the contrary, as shown in Ref. [5], a rectangular field of 6 BHEs, which operates for 30 years only in heating mode and without groundwater movement, undergoes a severe progressive temperature decrease. In this case, the ground needs a very long rest period (70 years) to recover its initial undisturbed temperature distribution.

In Ref. [6], the long-term sustainability of large BHE fields in the absence of groundwater flow is investigated numerically. It is found that for a single line or two staggered lines of infinite BHEs at least a partial compensation of winter heating and summer cooling is needed; on the other hand, an almost complete compensation of winter and summer loads is mandatory for a square field of infinite BHEs.

In Ref. [7], the long term performance of double U-tube borehole heat exchanger fields, placed in a porous ground with groundwater movement, is studied numerically with reference to a single line of infinite BHEs, two staggered lines of infinite BHEs and four staggered lines of infinite BHEs. The results show that the groundwater movement, even with a very low speed (a few mm per day), has an important effect on the sustainability of large BHE fields.

In Ref. [8], the Authors determine an analytical solution for a line heat source with constant power per unit length placed in an infinite porous medium with a uniform groundwater velocity. Since, under the assumption of uniform velocity, the problem is linear, the solution can be used to determine the temperature field around a set of line heat sources, by means of the superposition principle.

However, when the cylindrical heat source model is adopted, the groundwater velocity is no more uniform, the problem becomes non-linear and the superposition principle seems no more applicable.

In the present paper, the software COMSOL Multiphysics is used to investigate the long-term performance of BHE fields, where each BHE is sketched as a cylindrical heat source interested by a uniform time dependent heat flux with a partial compensation between winter and summer loads. The soil is modelled as a Darcy porous medium and the effects of groundwater movement are relevant. Aim of the paper is to propose and validate a simplified model to determine the long-term time evolution of the mean surface temperature of each BHE in a field, produced by a regular, properly time averaged, heat load.

2. Combined heat transfer model

Let us consider double U-tube BHEs placed in a soil that is modelled as a water-saturated porous medium for which the Darcy law holds. Let us study the BHE field performance over a period of 50 years by means of the cylindrical heat source model. Accordingly, the real 3D heat transfer problem can be reduced to a 2D unsteady model of the BHE cross-section. Moreover, let us assume that each BHE of the field is subjected to the following unbalanced time periodic heat load:

$$Q = \frac{3}{4} Q_0 \sin\left(\frac{2\pi}{\tau_0}\tau\right) + \frac{1}{4} Q_0 \left|\sin\left(\frac{2\pi}{\tau_0}\tau\right)\right| \quad , \quad (1)$$

where $\tau_0 = 3.1536 \cdot 10^7\,\mathrm{s}$ is the period of one year, and $|Q_0|$ is the highest magnitude of the (regular, time averaged) heat load per unit BHE length in each year (typical values are about 30 W/m). We assume that Q_0 is negative for winter leading operation (main heat quantity extracted from the ground) and positive in the opposite case of leading summer operation.

The mass, momentum and energy balance equations are

$$\nabla \cdot \vec{u} = 0 \ , \qquad (2)$$

$$\frac{\mu_w}{K}\vec{u} = -\nabla p \ , \qquad (3)$$

$$\rho_w c_w \left(\sigma \frac{\partial T}{\partial \tau} + \vec{u} \cdot \nabla T\right) = k_g \nabla^2 T \ , \qquad (4)$$

where $\vec{u}$ is the groundwater velocity vector, p is the pressure, T is the temperature, τ is time; ρ_w, c_w and μ_w are the density, the specific heat capacity and the dynamic viscosity of groundwater, K is the permeability of the porous medium, k_g is the effective thermal conductivity of the ground, σ is the ratio between the effective heat capacity per unit volume of the ground and that of groundwater,

$$\sigma = \frac{\rho_g\, c_g}{\rho_w\, c_w} \ . \qquad (5)$$

Let us define the following dimensionless quantities:

$$x^* = \frac{x}{D} \ , \ y^* = \frac{y}{D} \ , \ \tau^* = \frac{k_g}{\rho_w c_w D^2}\tau \ , \ Q^* = \frac{Q}{Q_0} \ ,$$

$$\psi^* = \frac{\rho_w c_w}{k_g}\psi \ , \ T^* = \frac{(T - T_g)\, k_g}{Q_0} \ , \qquad (6)$$

where D is the BHE diameter, T_g is the undisturbed ground temperature and ψ is the stream function, defined as

$$u = \frac{\partial \psi}{\partial y} \ , \ v = -\frac{\partial \psi}{\partial x} \ , \qquad (7)$$

u and v being the x and y component of $\vec{u}$, respectively. Equation (7) makes the mass balance equation (2) identically satisfied. By means of Eq. (6), the momentum and energy balance equations can be written in the dimensionless form

$$\nabla^{*2}\psi^* = 0 \ , \qquad (8)$$

$$\sigma \frac{\partial T^*}{\partial \tau^*} + \frac{\partial \psi^*}{\partial y^*}\frac{\partial T^*}{\partial x^*} - \frac{\partial \psi^*}{\partial x^*}\frac{\partial T^*}{\partial y^*} = \nabla^{*2} T^* \ , \qquad (9)$$

where ∇^{*2} is the dimensionless Laplace operator.

Let us define modified Peclet and Fourier numbers as

$$Pe = \frac{\rho_w c_w W D}{k_g} \ , \ Fo = \frac{k_g \tau_0}{\rho_w c_w D^2} \ , \qquad (10)$$

where W is the magnitude of the undisturbed groundwater velocity. Then, the dimensionless boundary conditions on the computational domain are as follows:

$$\frac{\partial \psi^*}{\partial x^*} = Pe \ , \ T^* = 0 \ , \qquad (11)$$

at the inlet side of the groundwater flow;

$$\frac{\partial \psi^*}{\partial y^*} = 0 \ , \ \vec{n} \cdot \nabla^* T^* = 0 \ , \qquad (12)$$

where $\vec{n}$ is the outward normal unit vector and $\nabla^* = \left(\frac{\partial}{\partial x^*}, \frac{\partial}{\partial y^*} \right)$, at the sides parallel to the groundwater flow;

$$\frac{\partial \psi^*}{\partial x^*} = Pe \ , \ convective \ \ flux \ , \qquad (13)$$

at the exit side of the groundwater flow;

$$\left(\frac{\partial \psi^*}{\partial y^*} ; - \frac{\partial \psi^*}{\partial x^*} \right) \cdot \vec{n} = 0 \ , \qquad (14)$$

$$\vec{n} \cdot \nabla^* T^* = -\frac{1}{\pi} \left[\frac{3}{4} \sin\left(\frac{2\pi}{Fo} \tau^* \right) + \frac{1}{4} \left| \sin\left(\frac{2\pi}{Fo} \tau^* \right) \right| \right]$$

at the interface between BHEs and ground.

The following BHE fields are studied: a single line of three BHEs; a square field of twenty-five BHEs. The BHE lines are assumed to be orthogonal to the undisturbed groundwater velocity and the distance between two adjacent BHEs is assumed equal to 40 D (for a typical double U-tube BHE, one has $D \cong 0.15$ m and, thus, a distance of about 6 m). A rectangular computational domain having a width equal to 4000 D and a height equal to 6000 D is adopted in the simulations. A sketch of the considered domain is shown in Fig.1a for the single line of three BHEs and in Fig.1b for the square field of twenty-five BHEs; the symmetry of the BHE field with respect to the y axis has been used in order to reduce the domain size and, thus, the computational effort.

The independence of the results of both the domain size and the mesh size has been checked; for the final computations an unstructured mesh made of 43636 triangular elements has been adopted for both cases.

For each BHE field, the following values of the dimensionless parameters are used in the simulations: $\sigma = 0.7$; $Pe = 0.05$; 0.4; $Fo = 300$; 550; 800. Therefore, 12 different cases are simulated. Simulations are performed by considering 50 periods, *i.e.* $0 \le \tau^* \le 50\ Fo$, with a dimensionless time step equal to 2 Fo/365.

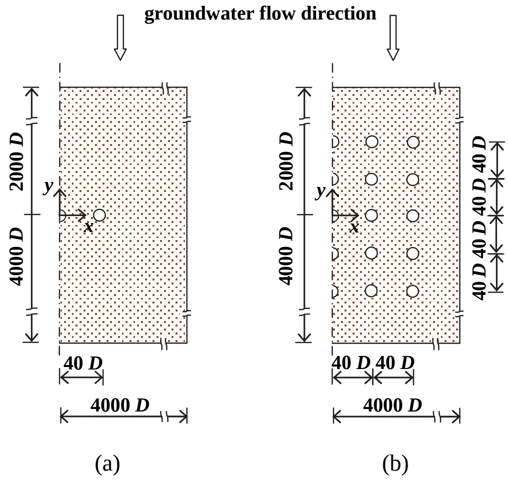

Figure 1. Computational domain: single line of 3 BHEs (Fig.1a); square field of 25 BHEs (Fig.1b).

The simulations of the BHE fields are performed by considering only the ground as computational domain, with BHEs replaced by holes. The dimensionless mean temperature T_s^* of the boundary surface of each hole (interface ground/grout) is evaluated as

$$T_s^*\left(\tau^*\right) = \frac{1}{\pi} \int_s T^* dl \ , \qquad (15)$$

where s is the BHE dimensionless boundary.

A simple method to evaluate the long-term performance of BHE fields would be to determine the temperature at the surface of a BHE by means of the superposition principle. However, since the BHEs are modelled by cylindrical heat sources, the groundwater velocity is non uniform and the superposition principle does not hold. Indeed, since the distance between adjacent BHEs is much higher than the BHE diameter, the velocity distribution is nearly uniform, except in very small portions of the domain. Thus, one can argue that the superposition method can be applied, with an acceptable accuracy.

Thus, with reference for instance to a line of 3 BHEs, the dimensionless temperature T_s^*

on the surface of the central BHE can be determined as the sum of the dimensionless temperature on the surface of the BHE placed in $x^* = 0$ and two times the dimensionless temperature on the surface of the BHE placed in $x^* = 40$. In this way, a single simulation performed by means of COMSOL Multiphysics on a single BHE can allow the determination of the performance of any BHE field, provided that one evaluates the temperature on each surface where the BHEs in the real field are found.

In the following section, the simplified method proposed is tested by comparing the results with those obtained by means of a complete BHE field simulation. In detail, reference is made to the long-term time evolution of the mean surface temperature of a specific BHE: the central BHE for the line of three BHEs and the central BHE of the last row (with respect to the groundwater flow direction) for the square field of twenty-five BHEs.

3. Results

The simulations performed show that the groundwater movement has an important effect on the long-term performance of the considered BHE fields. Indeed, the highest value of T_s^* is smaller for $Pe = 0.4$ than for $Pe = 0.05$. With reference to the case $Fo = 550$, the time evolution of T_s^* for the central BHE of the line of 3 BHEs is shown in Fig.2, whereas the plot of $T_s^* = T_s^*(\tau^*)$ for the central BHE in the last row of the square field of 25 BHEs is reported in Fig.3.

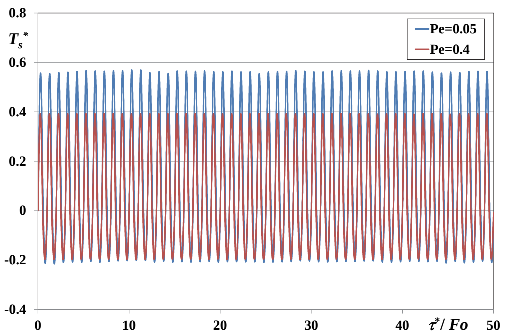

Figure 2. Line of 3 BHEs: time evolution of T_s^* for the central BHE, in the case $Fo = 550$.

As stressed above, the main aim of the simulations is to validate the proposed simplified method to determine the long-term performance of BHE fields.

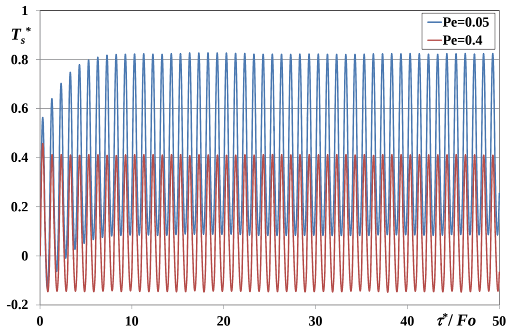

Figure 3. Square field of 25 BHEs: time evolution of T_s^* for the central BHE of the last row, in the case $Fo = 550$.

Figure 4 shows a comparison between the plot of $T_s^*(\tau^*)$ obtained by simulating the whole BHE field and that obtained by applying the simplified method (simulation of a single BHE); reference is made to the central BHE of the line of three BHEs, in the case $Pe = 0.4$, $Fo = 550$. For a clearer comparison, only the first two dimensionless time periods of simulations are represented. The figure shows that an excellent agreement is found between the results.

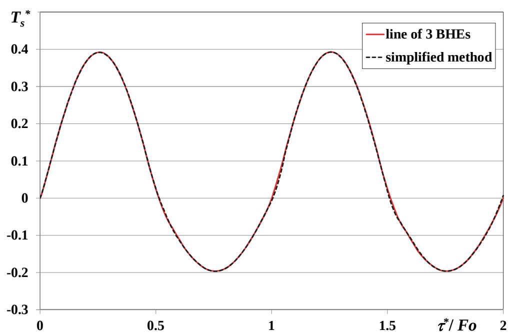

Figure 4. Plot of T_s^* for the central BHE of a line of 3 BHEs, obtained by simulating the whole field (red line) and by applying the simplified method (dashed black line), in the case $Pe = 0.4$, $Fo = 550$.

Figure 5 presents a comparison between the plot of $T_s^*(\tau^*)$ obtained by simulating the whole BHE field and that obtained by applying the simplified method, with reference to the central BHE of the last row of a square field of 25 BHEs, for $Pe = 0.4$, $Fo = 550$. Also in this case, the simplified method allows to determine $T_s^*(\tau^*)$ with a very good accuracy.

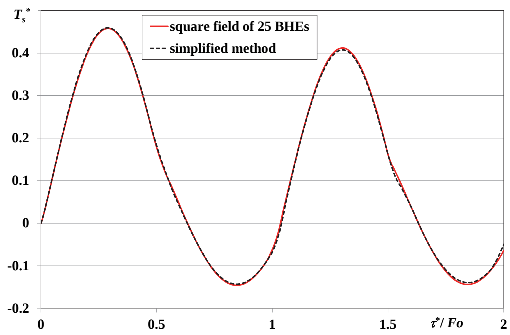

Figure 5. Plot of T_s^* for the central BHE of the last row of a square field of 25 BHEs, obtained by simulating the whole field (red line) and by applying the simplified method (dashed black line), for *Pe* = 0.4, *Fo* = 550.

In order to estimate the accuracy of the proposed simplified method, one can define the following parameter

$$\text{RSS\%} = \frac{\sum_{\tau^*/Fo=0}^{50}\left(T^*_{s\ s.m.} - T^*_{s\ BHE\ field}\right)^2}{\left|\left(T^*_{s\ BHE\ field}\right)_{\max}\right|}\cdot 100\ . \quad (16)$$

The numerator, in Eq. (16), is the sum of squared residuals (RSS) between T_s^* evaluated by the simplified method (subscript *s.m.*) and T_s^* evaluated by simulating the whole field (subscript *BHE field*); the denominator is the highest value of $T^*_{s\ BHE\ field}$. The parameter RSS% is a measure of the discrepancy between the dimensionless temperature determined by the simulation of the BHE field and that determined by the simplified method.

The values of RSS% are reported in Tables 1 and 2, for any considered pair of values (*Pe*, *Fo*), with reference to a line of 3 BHEs and to a square field of 25 BHEs, respectively. The Tables show that RSS% is smaller than 2% for the line of 3 BHEs, and smaller than 5% in the more critical case of a square field of 25 BHEs. Thus, the proposed simplified method can be considered as sufficiently accurate for design purposes.

A further simplification of the method consists in considering the dimensionless temperature on the BHE axis instead of the mean value over the BHE surface. In detail, with reference for instance to the line of 3 BHEs, the dimensionless temperature T_s^* on the surface of the central BHE can be determined as the sum of the dimensionless temperature on the surface of the BHE placed in $x^* = 0$ and two times the dimensionless temperature on the axis of the BHE placed in $x^* = 40$. In this way, the post processing computational time is significantly shortened since only one surface integrals of T^* must be evaluated.

Table 1. Values of RSS% and RSSa% for the line of 3 BHEs.

Line of 3 BHEs				
Pe	*Fo*	300	550	800
0.05	RSS%	1.3940	0.9787	1.0759
	RSSa%	1.3940	0.9789	1.0762
0.4	RSS%	0.7971	0.8382	0.7714
	RSSa%	0.7971	0.8382	0.7714

Table 2. Values of RSS% and RSSa% for the square field of 25 BHEs.

Square Field of 25 BHEs				
Pe	*Fo*	300	550	800
0.05	RSS%	4.5639	1.3421	1.7990
	RSSa%	4.5639	1.3421	1.7991
0.4	RSS%	0.9729	1.2708	1.0317
	RSSa%	0.9729	1.2720	1.0329

The values of RSS% evaluated by considering this further simplification in the method are denoted by RSSa% and are reported in Tables 1 and 2. The Tables show that RSSa% is practically coincident with RSS% . Therefore, the further simplification is recommendable, because it does not introduce additional errors.

4. Conclusions

The long-term performance of double U-tube borehole heat exchanger fields, placed in a porous ground with groundwater movement, has been studied, in dimensionless form, by finite element simulations performed through the software package COMSOL Multiphysics. The BHEs have been modelled as cylindrical heat sources subjected to a heat load with partial compensation of winter heating and summer cooling. A line of 3 BHEs and a square field of 25 BHEs have been considered.

A simplified method to determine the long-term time evolution of the mean surface temperature of each BHE in a field is proposed; the method is based on the superposition principle. The accuracy of the simplified method has been proved to be very good, so that the method can be conveniently adopted to evaluate the long-term performance of any BHE field.

5. References

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