

Designing Materials for Mechanical Invisibility Cloaks

Master's thesis in Solid and Structural Mechanics

Agenda

I. Introduction

- ❖ *Background*
- ❖ *Purpose*
- ❖ *Limitations*

II. Modeling of meta-materials

- ❖ *Definition*
- ❖ *Computational Homogenization*
- ❖ *Micro-polar material*

III. Mechanical cloaking of structures

IV. FE- implementation

- ❖ *MatLab*
- ❖ *COMSOL Multiphysics™*

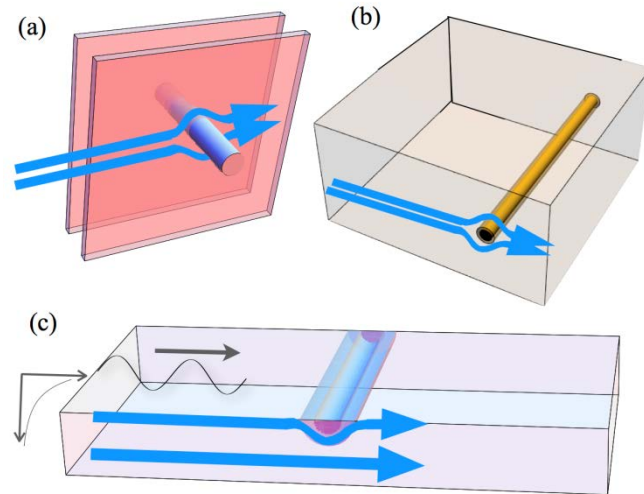
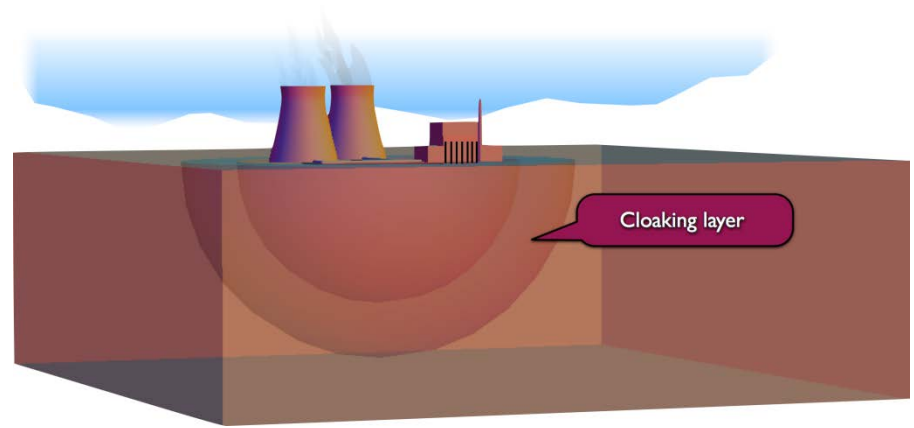
V. Results

- ❖ *Geometry*
- ❖ *Mesh*
- ❖ *Simulation results*

VI. Conclusions

Introduction

- Background



Modeling of meta-materials

- Definitions

- *Artificial material*

- Computational Homogenization

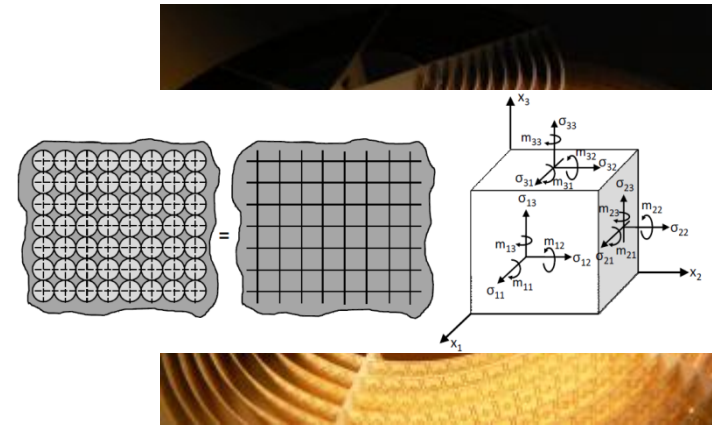
- Micro-polar material

- *Constitutive equation*
- *Equation of motion*
- *Assumptions*

- Centro-symmetric, $\mathbf{B} = \mathbf{0}$
- Time-harmonic conditions, $\exp(i\omega t)$
- Curvature stiffness is much higher than the stiffness with respect to strain

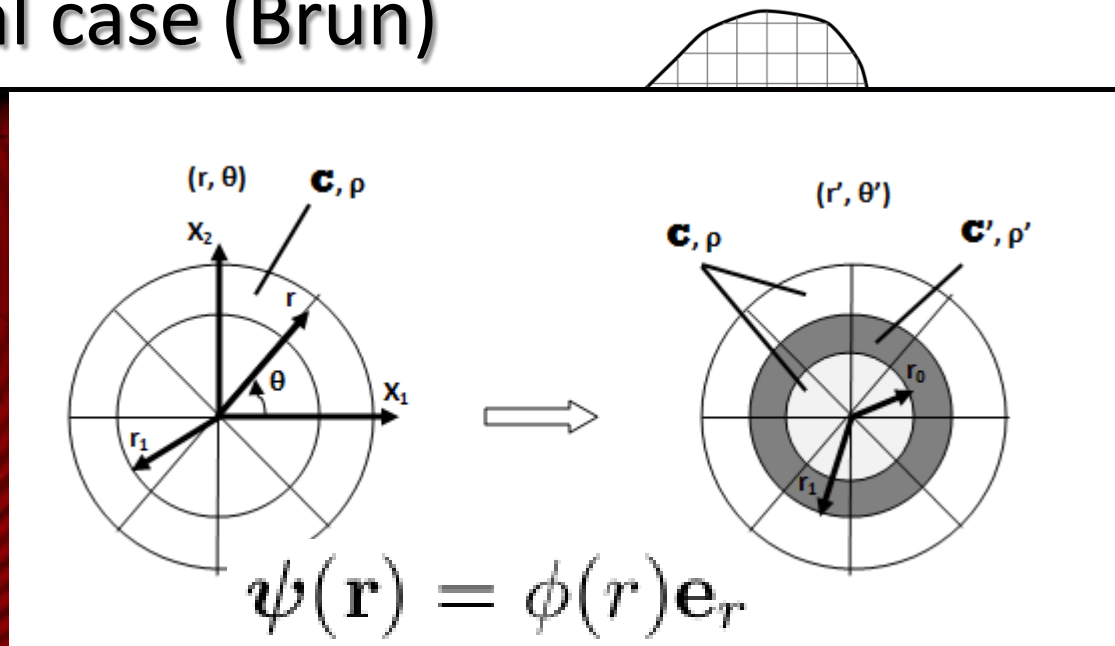
$$\nabla \cdot \boldsymbol{\sigma}^T + \rho \omega^2 \mathbf{u} = \mathbf{0} \quad \text{in } \Omega$$

$$\boldsymbol{\sigma} = \mathbf{C} : (\nabla \otimes \mathbf{u})$$



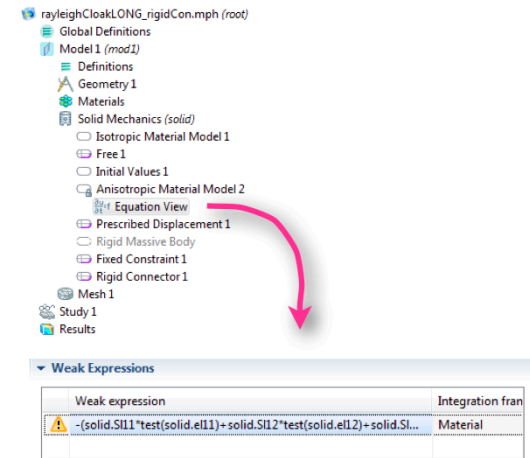
Mechanical cloaking of structures

- Cloaking transformation
- Special case (Brun)



FE-implementation

- Matlab etc.
- COMSOL Multiphysics™
 - *Modifications of the software*

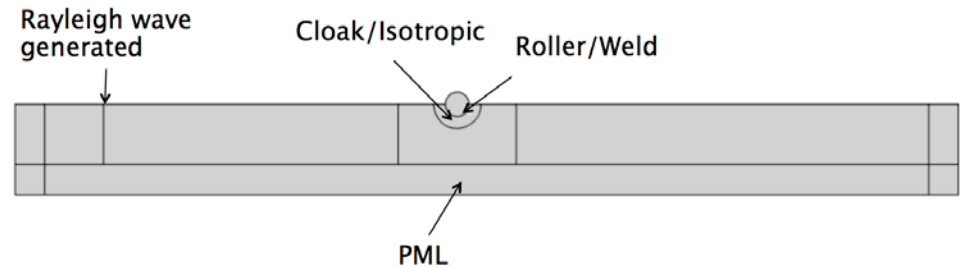


Name	Expression
C1111	$((\lambda + 2\mu) \cdot f) / (s \cdot f)$
C2222	$((\lambda + 2\mu) \cdot s \cdot f) / f$
C1122	λ
C2211	λ
C1221	μ
C2112	μ
C1212	$(\mu \cdot f) / (s \cdot f)$
C2121	$(\mu \cdot s \cdot f) / f$

Results

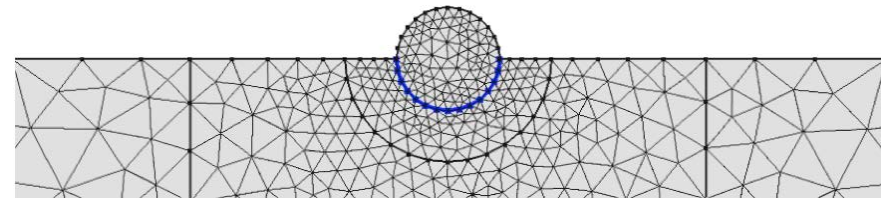
- **Geometry**

- *Considered case*
- *Implementation in COMSOL Multiphysics™*



- **Mesh**

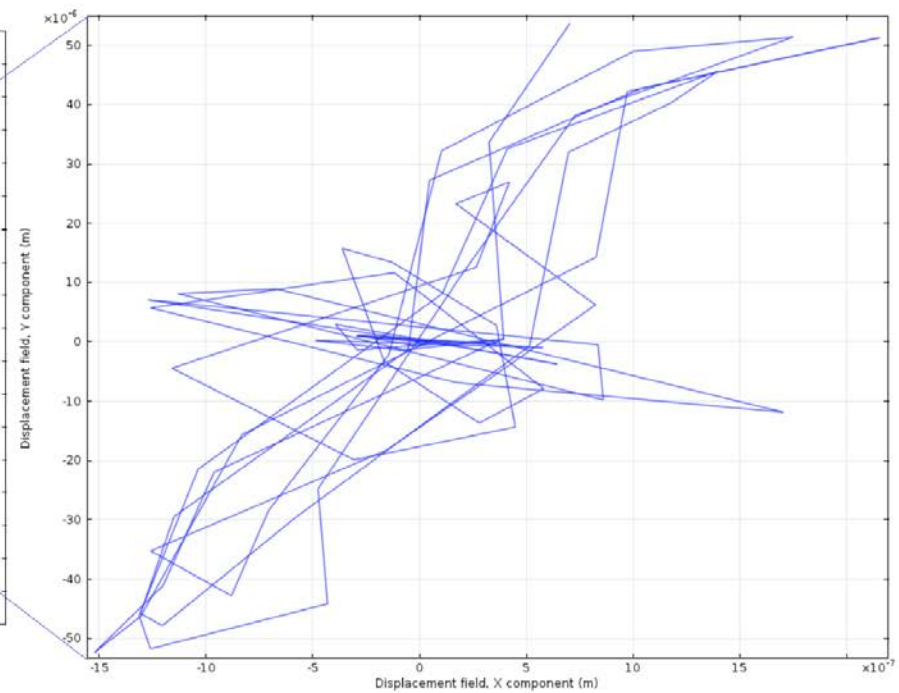
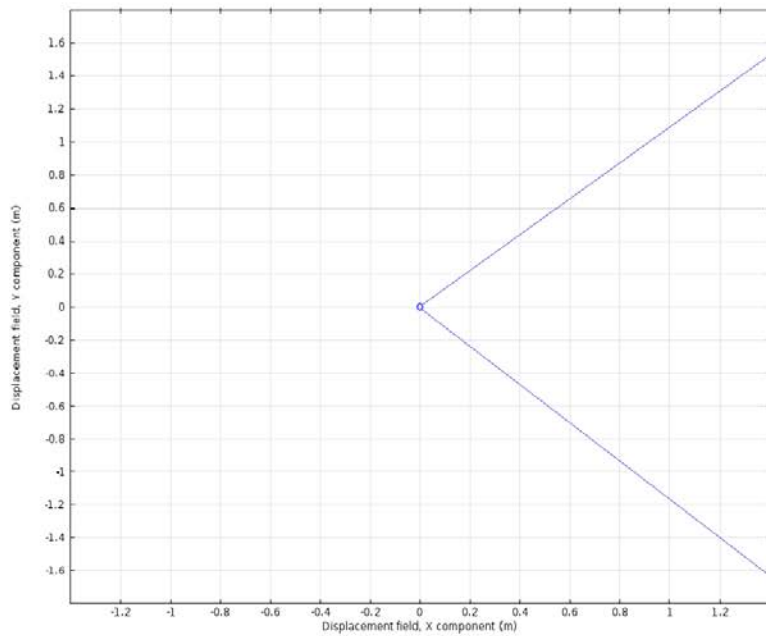
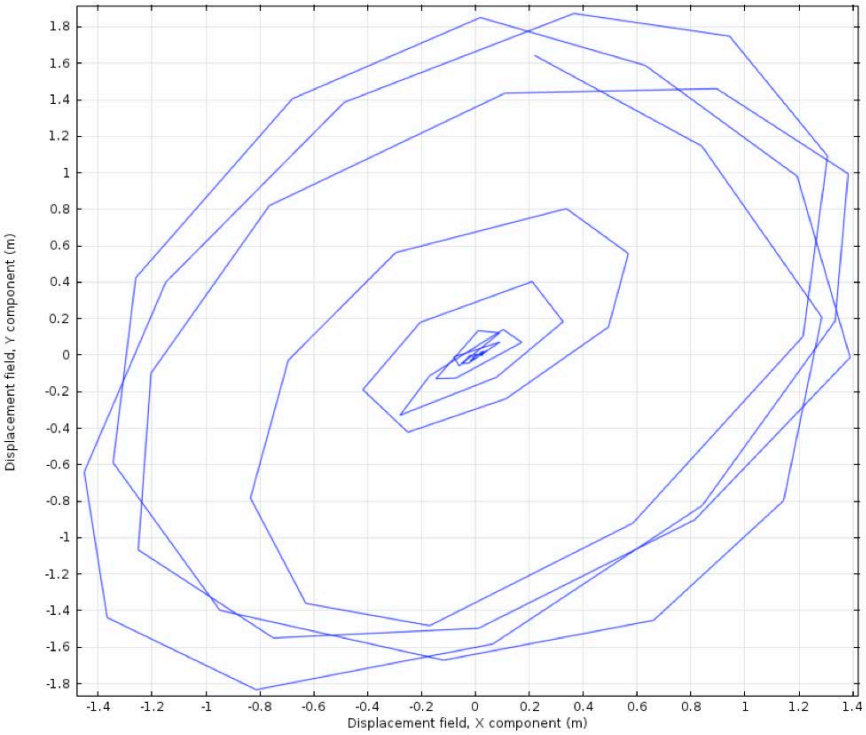
- *Mesh size vs. Time-step*

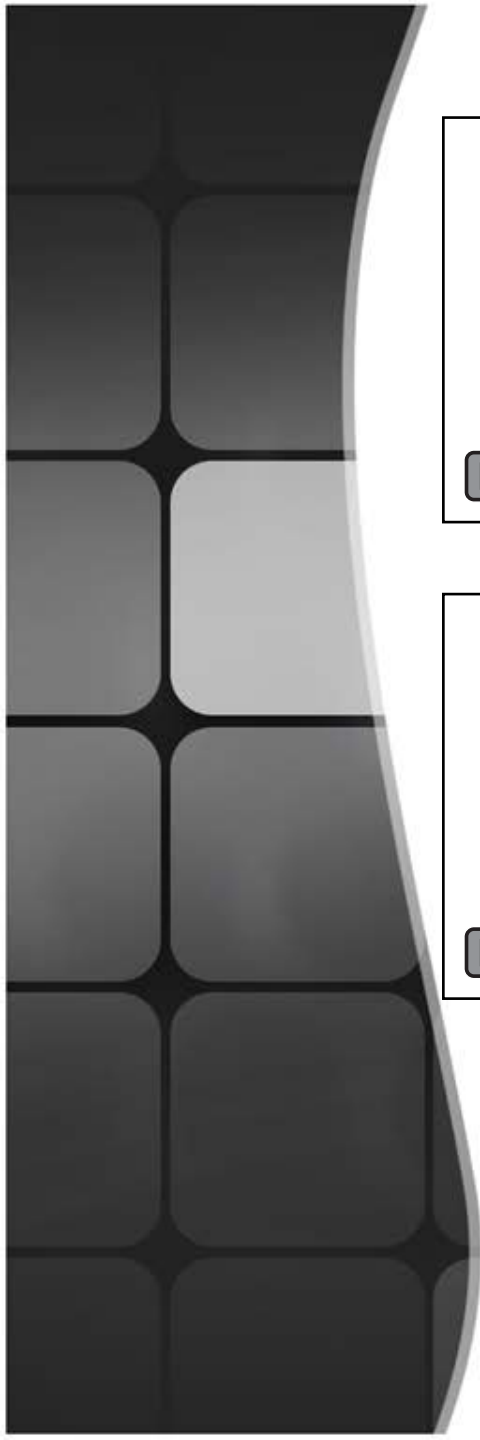


- **Simulation results**

- *Displacement*
- *Example – Rayleigh waves*

$$\Delta t \leq \frac{L}{C_s}$$





Conclusions

- Real life applications
- Difficulties & Error sources
- Future work

