Single Phase Flow Models in Fractal Porous Media Using a Fractal Continuum Mechanics Approach

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Introduction: The primary motivation of this work was to develop mathematical and numerical models for fluid flow in porous media with fractal properties, because it has been observed that the pressure from well tests of certain naturally fractured reservoirs in México exhibit an abnormal behavior which departs from the expected when traditional flow models with Euclidean geometry are applied.

Some authors like (Camacho-Velázquez et. al. 2006), (Barker 1988), (Chang and Yortsos 1990), among many others, believe that the reason for this anomalous behavior can be explained assuming that the porous medium has fractal properties due to the complex distribution of fractures. Recently, (Tarasov 2005) and (Ostoja-Starzewski, et. al. 2011, 2013) have introduced fractional measures for isotropic and anisotropic fractal media, respectively, which allowed a systematic derivation of fractal flow models using a fractional continuum mechanics approach. The theory of fractional continuum mechanics can be interpreted as a generalization of the usual theory of continuum mechanics but introducing fractional measures instead of Lebesgue measure.

In this work, two models for single phase flow in porous media with fractal properties were derived to evaluate their performance numerically. One of the advantages of the resulting mathematical models of anomalous flow obtained in this work is that they are represented in terms of conventional differential equations in which their coefficients are functions of the fractal (mass and boundary) dimensions, i. e., fractional differential equations can be expressed as equations with integer derivatives, which has a great advantage for their numerical solution and especially for its computational implementation. Numerical results showed consistency with the expected anomalous behavior, where the pressure drops at a faster or slower rate compared to the conventional flow model.

Mathematical Models:

1. Single phase flow model for ***isotropic*** fractal porous media:

$$\phi c_t \frac{\partial p}{\partial t} - \nabla^D \cdot \left(\frac{1}{\mu}\underline{\underline{k}} \cdot \nabla^D p\right) = 0 \qquad c_2(D,r) = \frac{2^{D-2} r^{2-D}}{\Gamma(D/2)}; \qquad c_1(d,r) = 2^{1-d}\frac{\sqrt{\pi}\, r^{d-1}}{\Gamma(d/2)}$$

$$\nabla^D p \equiv c_2^{-1}(D,r)\nabla(c_1(d,r)\,p) \qquad c_2^{-1}(D,r) = \frac{1}{c_2(D,r)}; \qquad r = |\underline{x}-\underline{x}_0|, \quad \underline{x}_0 \in B(t)$$

Γ is the Gamma function

Initial condition: $p(t_0) = p_0$
Boundary conditions: $\underline{u}\cdot\underline{n} = 0$ (No-flow conditions at all boundaries.)

2. Single phase flow model for ***anisotropic*** fractal porous media:

$$\phi c_t \frac{\partial p}{\partial t} - \nabla \cdot \left(\frac{1}{\mu}\underline{\underline{k}} \cdot \tilde{\nabla}^D p\right) = 0 \qquad D = \alpha_1 + \alpha_2$$

$$\tilde{\nabla}^D p \equiv \left(\frac{1}{c_1^{(1)}}\frac{\partial p}{\partial x_1}, \frac{1}{c_1^{(2)}}\frac{\partial p}{\partial x_2}\right) \qquad c_1^{(k)} = \frac{|x_k|^{\alpha_k - 1}}{\Gamma(\alpha_k)}, \; k = 1,2 \text{ (no sum).}$$

Initial condition: $p(t_0) = p_0$
Boundary conditions: $\underline{u}\cdot\underline{n} = 0$ (No-flow conditions at all boundaries.)

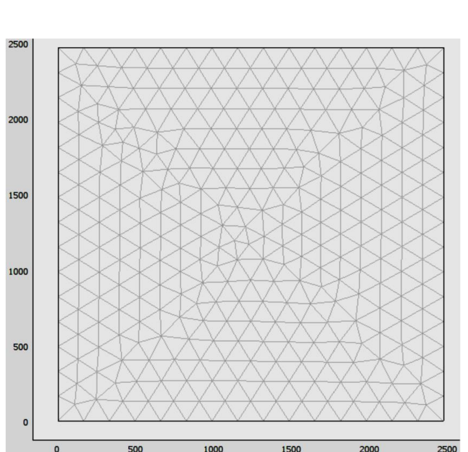

Figure 1. Title of the figure

Item	Description and (value)
p_0	Initial pressure (3600 psi)
μ	Oil viscosity (1.06 cP)
K	Permeability (0.3 Darcy)
x_0	x-coordinate of the well (1234.44 m)
y_0	y-coordinate of the well (1234.44 m)
c_f	Oil compressibility (0.00001 1/psi)
c_R	Rock compressibility (0.000004 1/psi)
c_t	Total compressibility (0.000014 1/psi)
ϕ	Porosity (0.2)
Q_0	Oil production rate (300 STB/D)

Table 1. Title of the table

Results: In **Figure 2**, it can be observed that the pressure in the well drops faster at the beginning but quickly stabilizes in a constant value with the increasing of the fractal boundary dimension for a fixed value of mass fractal dimension (*D=2*). Note that the conventional model (*D = 2 and d = 1*) is in red color. While in **Figure 3**, it is seen that the pressure behavior tends to be more linear as we move away from well with the increasing of the fractal boundary dimension. **Figure 4** shows that with the decrease of the mass fractal dimension (*D*) for a fixed fractal boundary value (*d = 1*) in the isotropic fractal model the behavior of the pressure drop in the well at the beginning is slower but later is faster becoming almost linear, which is very different in comparison with the conventional model (*D*=2 and *d*=1) in red color. While in **Figure 5**, it is seen that the pressure behavior tends to be lower but quickly stabilizes in a constant value as we move away from well with the decreasing of the mass fractal dimension.

In the anisotropic model (**Figures 6 and 7**), the behavior of the pressure drop in the well is similar but slower as the mass fractal dimension $D = \alpha_1 + \alpha_2$ increases in comparison with the conventional flow model. The behavior of the pressure around the well is symmetrical if the medium is isotropic $\alpha_1 = \alpha_2$ (see **Figure 8**) and asymmetric if the fractal medium is anisotropic $\alpha_1 \neq \alpha_2$ (see **Figure 9**) when the anisotropic fractal model is applied.

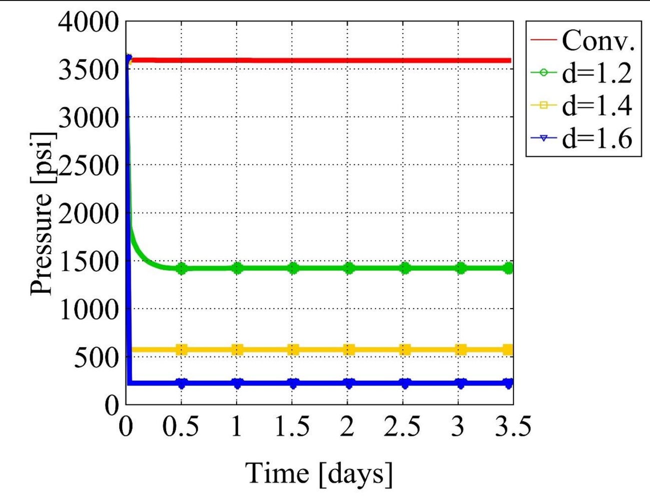

Figure 2. Pressure drop in the well during 3.5 days, for the isotropic fractal model with *D=2.0* and different values of the boundary dimension (*d*).

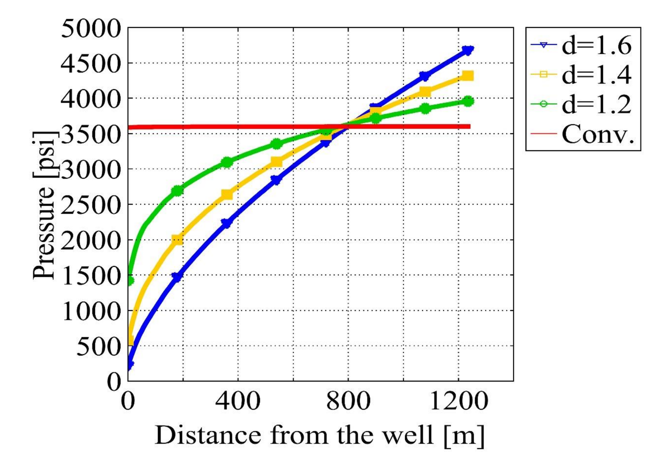

Figure 3 Pressure profile along a section for the isotropic fractal model with *D*=2.0 and different values of the fractal boundary dimension (*d*).

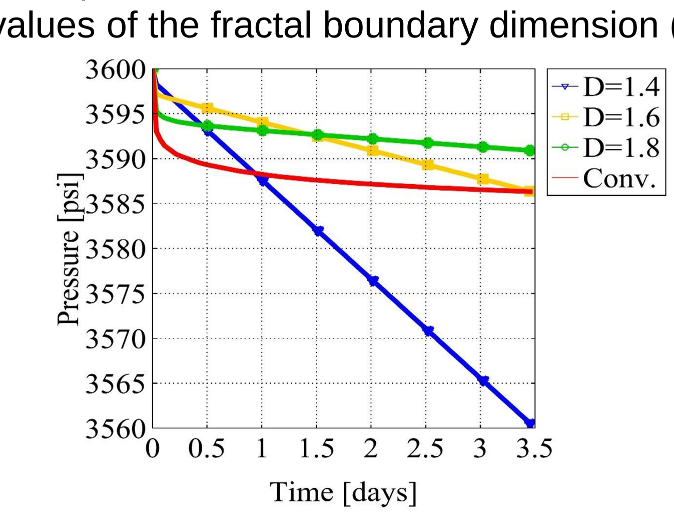

Figure 4 Pressure drop in the well during 3.5 days for the isotropic fractal model with *d*=1 and different values of mass fractal dimension (*D*).

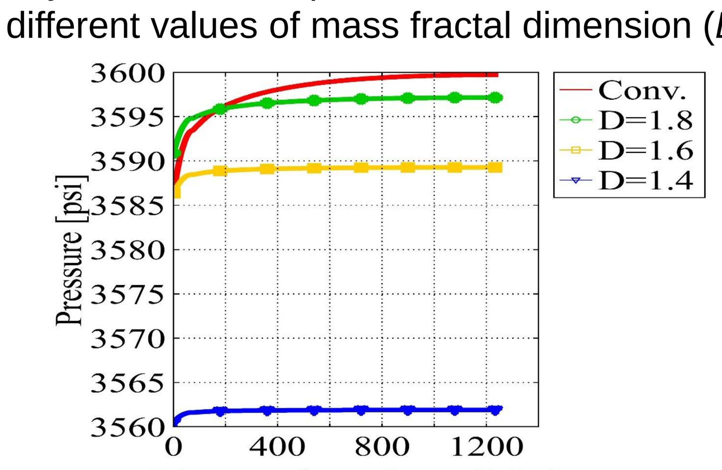

Figure 5 Pressure profile along a section for the isotropic fractal model with *d*=1 and different values of mass fractal dimension (*D*).

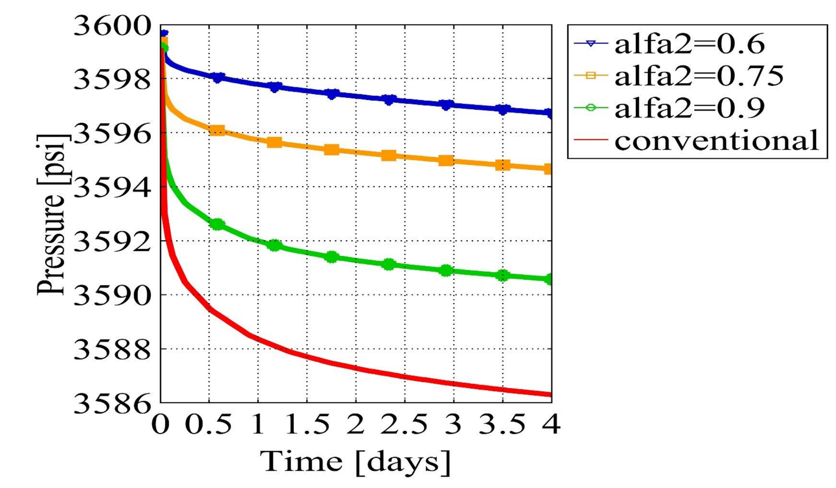

Figure 6 Pressure in the well during 4 days, with α_1=1 and different values of α_2 for the anisotropic fractal model.

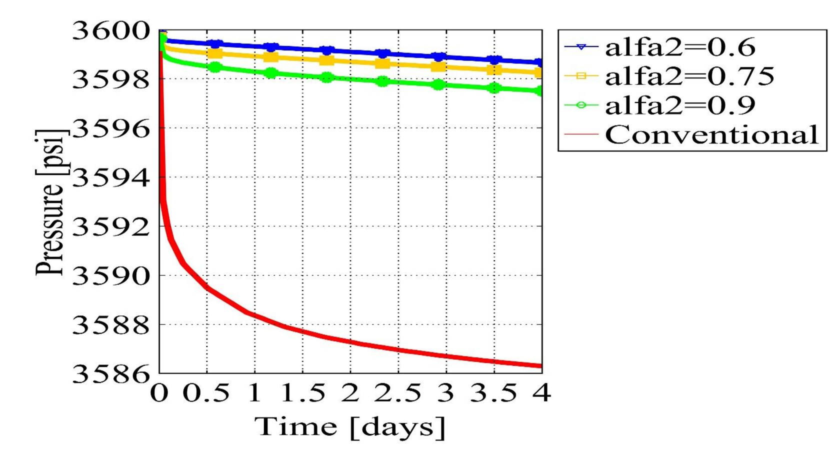

Figure 7 Pressure in the well during 4 days, with α_1=0.6 and different values of α_2 for the anisotropic fractal model.

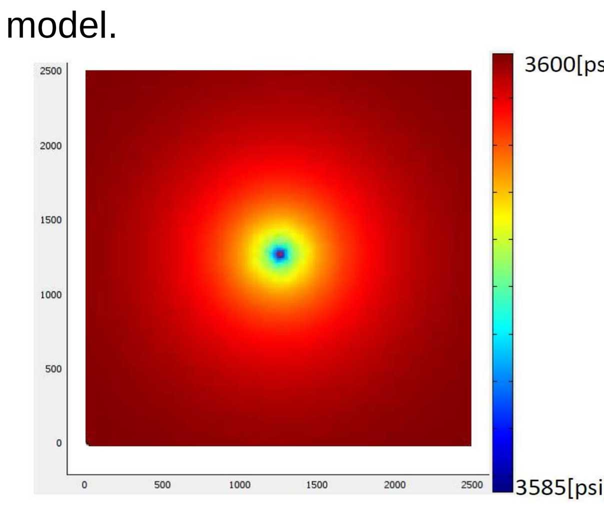

Figure 8 Pressure in 4 days for the anisotropic fractal model with α_1=0.6= α_2.

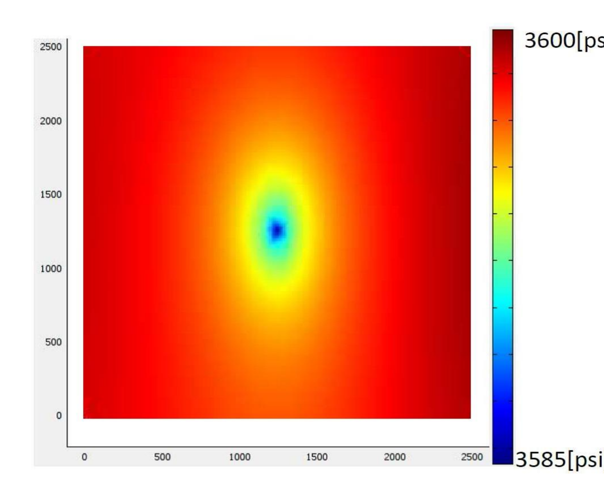

Figure 9 Pressure in 4 days for the anisotropic fractal model with α_1=0.75 and α_1=0.6 .

Conclusions: Applying a fractional continuum mechanics approach two single phase flow models in porous media with fractal properties were derived. The first one was developed for isotropic fractal media using a fractional measure introduced by (Tarasov 2005). Whereas the second model was obtained applying a fractional measure introduced by for (Ostoja-Starzewski, et. al. 2011 and 2013) for anisotropic media. Both models required unconventional Darcy laws for fractal media.

The numerical experiments showed a behavior consistent with the question of anomalous diffusion, where the pressure drops with faster or slower rate compared to conventional flow model. One of the advantages of the mathematical models of abnormal flow obtained in the present work is that they are conventional differential equations with additional numerical coefficients, i.e., fractional differential equations can be expressed in terms of integer derivatives, the latter being a great advantage for their numerical solution and computational implementation. The solutions of fractal flow models are reduced to the solution of the conventional model if the corresponding integer dimensions *(D=2 and d=1 or* $\alpha_1 = 1 = \alpha_2$*)* are taken.

Comparing the isotropic model with respect to the anisotropic model it can be seen that the first one in general depends on the parameters *(D, d,* $\underline{x}_0$*)* which is usually placed in the same position of the source term, while the second model only depends on the fractal dimensions in each direction (α_1, α_2). Moreover, the anisotropic model despite of being more general doesn't reduce to the first one. Therefore, both models constitute two alternatives for modeling flow in fractal porous media.

References:

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