1D central symmetry modelling of shrinkage for non-porous materials

Jean-Michel Meot[1], Aurélien Briffaz[1], Didier Jacquin[2,1], Salim Rashidi[2]
1 Cirad, UMR QualiSud, TAB 95/15, F-34398 Montpellier cedex 5
2 Bureau national Interprofessionnel du Pruneau, 2 Rue des Magnolias, F-47300 Villeneuve / Lot.

Introduction

Drying of high-water-content food products like fruits induces large deformations. The hypothesis of this work is that drying of a plum could be modelled in 1D central symmetry with conservation of water and dry matter volumes.

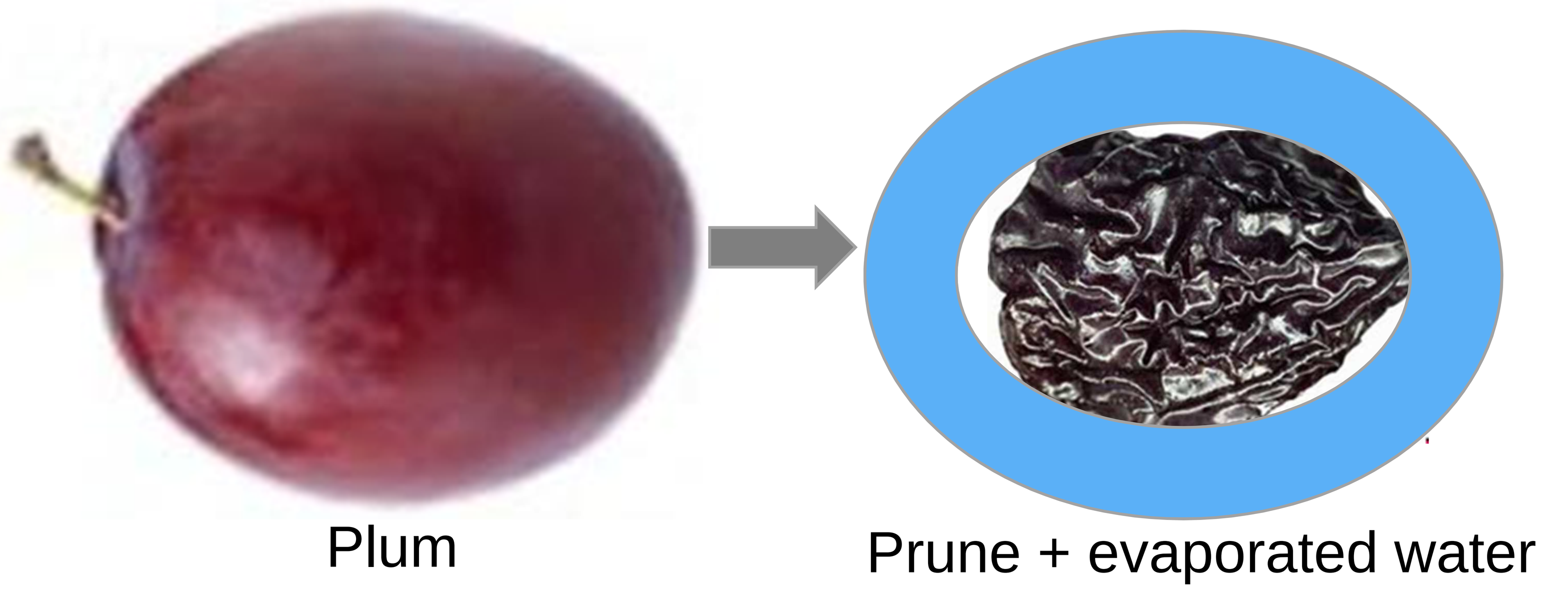

Figure 1. Schematization of plum drying process

Computational Methods

Heat transfer and water transport were expressed in Lagrangian coordinates as done by Briffaz et al. (2014) (1) using some physical parameters collected from the literature (2, 3). The model was implemented using "*coefficient form*" PDE mode. Solving of the governing heat transfer, water transport and deformation PDEs was carried out in the "*material frame*" which matches the initial volume of the fresh plum.

Dry matter (DM) conservation can be written as

$$\forall R \in [0\ R_{max}],\quad \int_0^R \rho_{DM,t}\, 4\,\pi\, r_t^2\, dr = \int_0^R \rho_{DM,0} 4\,\pi\, R^2\, dR$$

With *r* radius in spatial frame, *R* radius in material frame
$\rho_{DM,t}$ apparent density of dry matter in spatial frame at t

A solution is $\frac{d\left(\frac{4}{3}\pi r_t^3\right)}{dR} = 4\,\pi \frac{\rho_{DM,0}}{\rho_{DM,t}} R^2$ with r_t=*R* at *t=0*

Volume variations of the internal sphere with radius *R* ($\forall\ R \in [0\ R_{max}]$) are described by the following PDE

$$\nabla\left(\frac{4}{3}\pi r_t^3\right) = 4\pi \frac{\rho_{DM,0}}{\rho_{DM,t}} R^2 \text{with } \frac{4}{3}\pi r_0^3 = \frac{4}{3}\pi R^3$$

To describe local deformation, implementation of ALE has been made using the imposed deformation of a mobile mesh $dr_{0\to t} = -(R - r_t)$

Results

The model is mass-conservative with local deformation induced by local mass flux densities.
Water-content-interpolated water diffusivity gave promising fitting results with experimental drying data (figure 2).

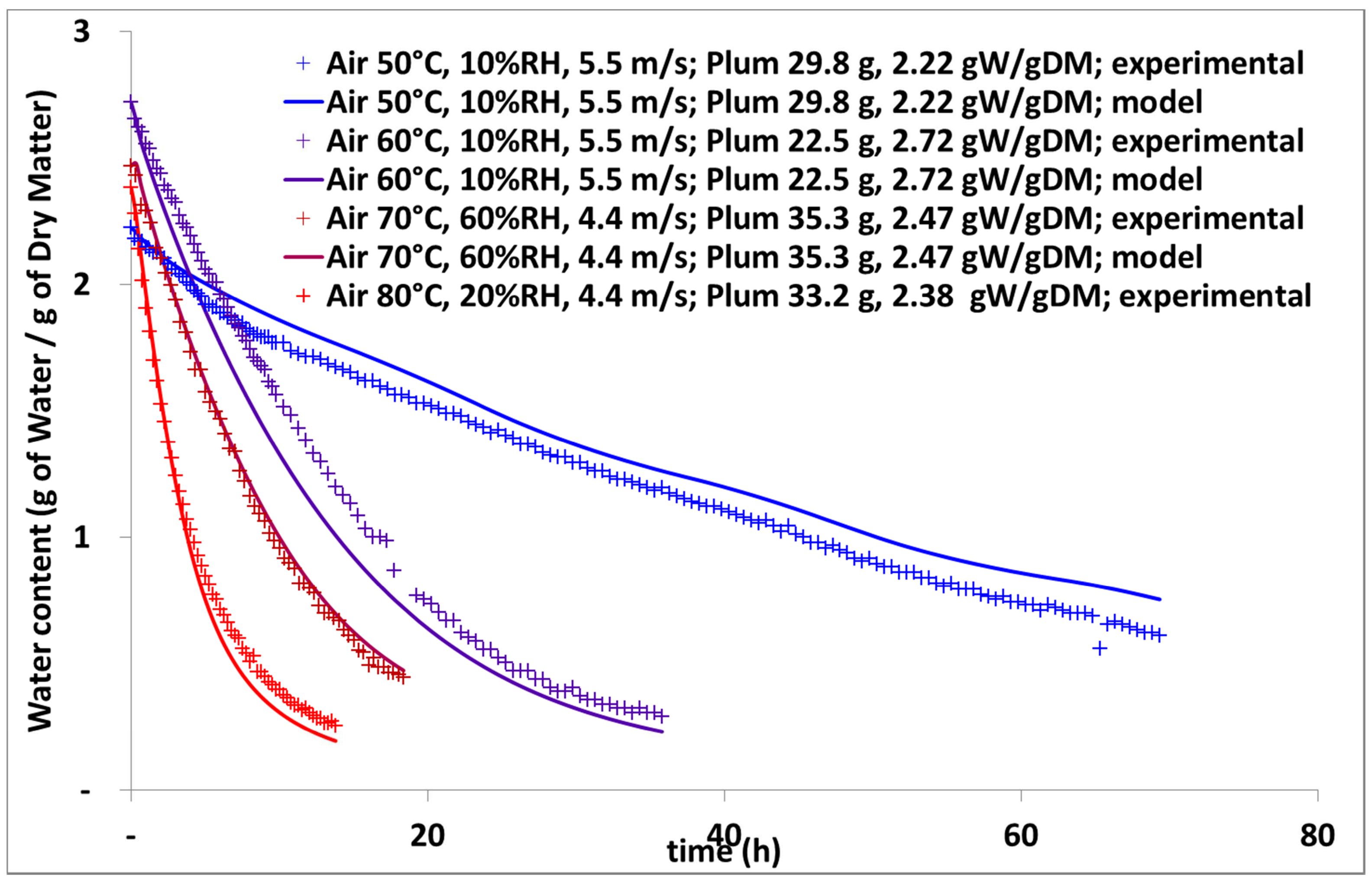

Figure 2. Experimental and model kinetics of plum drying

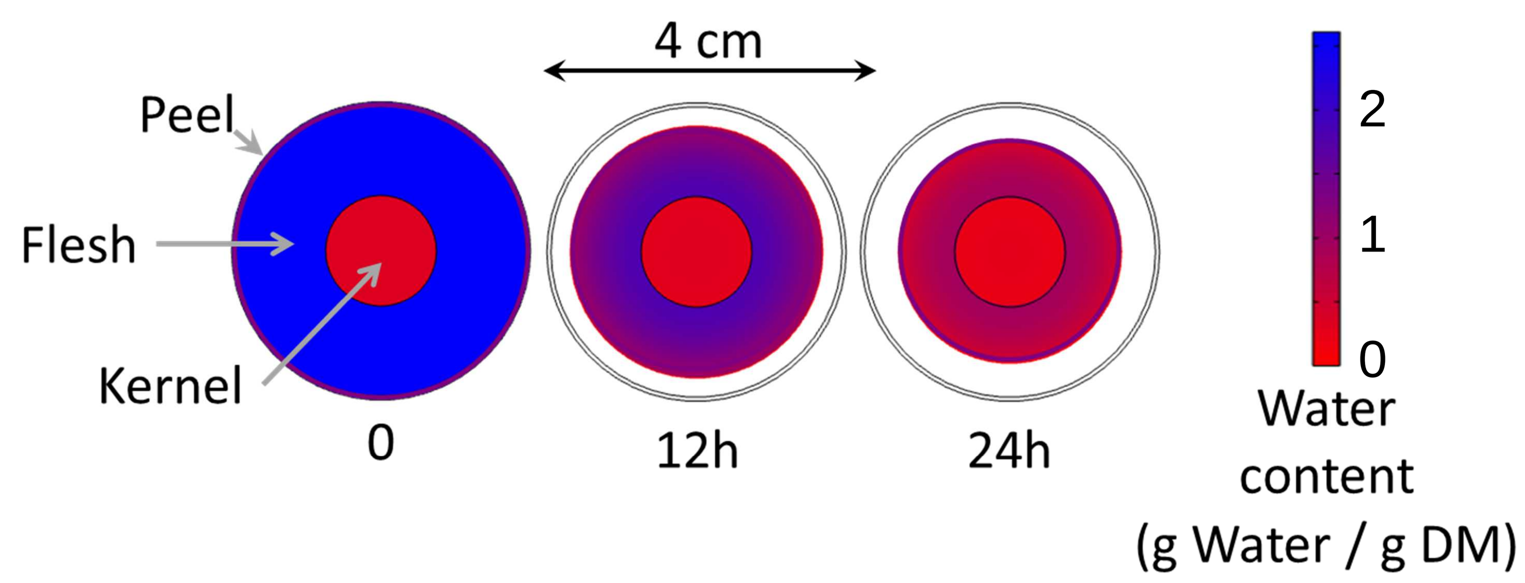

Figure 3. Spatial representation of shrinkage

Conclusions

This easy way to model shrinkage of non-porous materials is very useful for parameter identification, model validation, and communication with process supervisors. Nevertheless, it is limited to products that could be reduced to 1 dimension.

References:

1. A. Briffaz et. al., Modelling of water transport and swelling associated with starch gelatinization during rice cooking. Journal of food engineering, 121 : 143-151 (2014)
2. Tsami E., Marinos-Kouris D., Mroulis Z.B.; 1990; Water sorption isotherms of raisins, currants, figs, prunes and apricots.; Journal of food science 55(6), 1594 – 1625
3. H.T. Sabarez, Modelling of simultaneous Heat ad Mass transfert during drying of prunes, Acta Horiculturae 566, 421 - 428 (2001)