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Free convection in a square cavity partially filled with porous media with spatial wall temperature

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Convection

- The transfer of heat from one place to another by the movement of fluids.
- Convection can be "forced" by movement of a fluid or by natural buoyancy forces alone, when the fluid is heated (natural convection).
- A porous medium: a material consisting of a solid matrix with an interconnected void.
- The study of natural convection in cavity partially-filled with porous media has importance in many fields of science, engineering, chemical engineering and industrial applications.

Ground Water

Heated ground water due to hot intrusion may rise in a narrow fractured zone. As the heated water rises, it eventually encounters a cooler rock formation that sandwiches

the permeable vertically slender space. This causes heat transfer between the hot water and the colder surrounding rocks.



Partially-Filled Porous Medium

Solidification of castings

The biofilm growth







Heat Transfer in Cavity



Beavers and Joseph (1967) investigated the simple situation of the boundary conditions between a porous media and a homogeneous fluid.



Singh and Thorpe (1995) conducted a comparative study of different models for the investigation of natural convection in a confined fluid and overlying porous layer.



Saeid and Mohamad (2005) studied numerically the natural convection in a porous cavity with spatial sidewall temperature variation using finite element method.

Study the effect of spatial wall temperature on free convection in a square cavity partially filled with porous media has not been undertaken yet.

Mathematical Formulation

The conservation equations for mass, momentum and energy equations for the porous layer:



The conservation equations for mass, momentum and energy equations for the homogenous fluid layer are:

$$\frac{\partial u_f}{\partial x} + \frac{\partial v_f}{\partial y} = 0,$$

$$\begin{array}{c} \mathbf{6} \quad u_{f} \frac{\partial u_{f}}{\partial x} + v_{f} \frac{\partial u_{f}}{\partial y} = -\frac{1}{\rho_{f}} \frac{\partial p_{f}}{\partial x} + v \left(\frac{\partial^{2} u_{f}}{\partial x^{2}} + \frac{\partial^{2} u_{f}}{\partial y^{2}} \right), \\ \mathbf{7} \quad u_{f} \frac{\partial v_{f}}{\partial x} + v_{f} \frac{\partial v_{f}}{\partial y} = -\frac{1}{\rho_{f}} \frac{\partial p_{f}}{\partial x} v \left(\frac{\partial^{2} v_{f}}{\partial x^{2}} + \frac{\partial^{2} v_{f}}{\partial y^{2}} \right) + \rho g \beta \left(T_{f} - T_{c} \right), \\ \mathbf{8} \quad u_{f} \frac{\partial T_{f}}{\partial x} + v_{f} \frac{\partial T_{f}}{\partial y} = \left(\frac{\partial^{2} T_{f}}{\partial x^{2}} + \frac{\partial^{2} T_{f}}{\partial y^{2}} \right). \end{array}$$

The non-dimensional variables are:

$$\begin{split} \Psi &= \frac{\Psi}{\alpha_m \varphi L}, \ \theta_p = \frac{T_p - T_c}{T_h - T_c}, \ \theta_f = \frac{T_p - T_c}{T_h - T_c}, \\ U &= \frac{Lu}{\alpha}, V = \frac{L_v}{\alpha}, X = \frac{x}{L}, Y = \frac{y}{L}. \\ T_h(y) &= \overline{T_h} + \varepsilon \left(\overline{T_h} - T_c\right) \left[1 - \cos \left(\frac{2\pi k y}{L}\right) \right]. \end{split}$$

The governing equations for the porous layer can be written as:

$$\begin{array}{c} 11 \qquad \frac{\partial^2 \Psi_p}{\partial X^2} + \frac{\partial^2 \Psi_p}{\partial Y^2} = -RaDa \frac{\partial \theta_p}{\partial X}, \\ \hline 12 \qquad \frac{\partial \Psi_p}{\partial X} \frac{\partial \theta_p}{\partial Y} - \frac{\partial \Psi_p}{\partial Y} \frac{\partial \theta_p}{\partial X} = \frac{\lambda_p}{\lambda_f} \left(\frac{\partial^2 \theta_p}{\partial X^2} + \frac{\partial^2 \theta_p}{\partial Y^2} \right). \end{array}$$

The governing equations for the fluid layer can be written as:

$$U_{f} \frac{\partial U_{f}}{\partial X} + V_{f} \frac{\partial U_{f}}{\partial Y} = -\frac{\partial P_{f}}{\partial X} + \left(\frac{\partial^{2} U_{f}}{\partial X^{2}} + \frac{\partial^{2} U_{f}}{\partial Y^{2}}\right),$$

$$U_{f} \frac{\partial V_{f}}{\partial X} + V_{f} \frac{\partial V_{f}}{\partial Y} = -\frac{\partial P_{f}}{\partial Y} + \left(\frac{\partial^{2} V_{f}}{\partial X^{2}} + \frac{\partial^{2} V_{f}}{\partial Y^{2}}\right) + \frac{Ra}{Pr}\theta,$$

$$U_{f} \frac{\partial \theta_{f}}{\partial X} + V_{f} \frac{\partial \theta_{f}}{\partial Y} = \frac{1}{Pr} \left(\frac{\partial^{2} \theta_{f}}{\partial X^{2}} + \frac{\partial^{2} \theta_{f}}{\partial Y^{2}}\right),$$

The dimensionless boundary conditions of Eqs. (11)-(15) are:

$$\Psi(0,Y) = 0, \quad \theta_p(0,Y) = 0.5 + \varepsilon [1 - \cos(2\pi kY)],$$

$$\Psi(1,Y) = 0, \quad \theta_f(1,Y) = -0.5,$$

$$\Psi(X,0) = \Psi(X,1) = 0, \quad \frac{\partial \theta(X,0)}{\partial Y} = \frac{\partial \theta(X,1)}{\partial Y} = 0,$$

At the interface by using the matching conditions:

$$U^{+} = -V^{-},$$

$$\frac{\partial V}{\partial X} = \overline{\alpha}(U^{+} - V^{-}) / \sqrt{Da}$$

$$\theta|_{X=s^{+}} = \theta|_{X=s^{-}},$$

$$\lambda_{p} \frac{\partial \theta}{\partial X} s^{+} = \lambda_{f} \frac{\partial \theta}{\partial X} s^{-}.$$

The local Nusselt number along the hot and the cold walls, which are defined, respectively, by:

,

(18)
$$Nu_{h} = \frac{hL}{\lambda} = -\left(\frac{\partial\theta}{\partial X}\right)_{X=0}$$

(19) $Nu_{c} = \frac{hL}{\lambda} = -\left(\frac{\partial\theta}{\partial X}\right)_{X=1}$

Finally, the average Nusselt number can be defined based on the average heat transfer coefficient and is given by:

$$20 \quad \overline{Nu} = \frac{\overline{hL}}{\lambda} = \int_0^1 N \, u \, dY \, dY$$

Numerical Method and Validation

Galerkin finite element method (GFEM), the governing equations subject to the boundary conditions are solved numerically using the CFD software package COMSOL Multiphysics.

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Validation



Results

$$Ra = 10^7$$
, $Da = 10^{-4}$, $\varepsilon = 0.5$ and $S = 0.5$.

It is found that when the wave number increases, the expansion of the streamline circulation cell tend to increases horizontally. The strength of the flow circulation increases with increasing k value up to 2.5.

The isotherm patterns are raised with high intensity and with irregular-shaped next to the left wall by the increase of the wave number, while near to the cold wall, the isotherm patterns occur with vertical lines.



Heat Transfer Rate

 $Da = 10^{-4}$, k = 2.5 and $\varepsilon = 0.5$

 $Ra = 10^5$, $\varepsilon = 0.5$ and S = 0.5



Increasing Rayleigh number leads to increase the average Musselt number, due the fact that the fluid has higher thermal conductively than porous, the smaller porous thickness layer has stronger effect on the heat transfer rate which has higher average Nusselt number.

Future Works

3D Chaotic convection Turbulence flow Multiphase flow



THANK YOU