



Calculation Of The Helmholtz Resonator Using COMSOL® In An Acoustic Lecture



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Introduction

In the field of electronics, electro-acoustic apparatus is one of the important applications. There is an acoustic class in the department of electronic engineering, Shibaura Institute of Technology. To foster the student's comprehension on frequency characteristics of a cavity and on an acoustic element, the students will learn numerical analysis of a Helmholtz resonator. Generally, a resonance frequency of Helmholtz resonator is calculated by acoustic element concept. In the class, a can bottle or a glass one is employed as the Helmholtz resonator. In the practical measurement, we prepare a simple FFT analyzer. Using the built-in microphone in a personal computer, they can measure emitted tone sound when they blow the mouth of the bottle. As a result, they can confirm the precision of their rough calculation result. After the rough calculation, they analyze the frequency response of the bottle they employed using the finite element application, "COMSOL". We expect that they would understand the validity of FEM in order to improve the precision of the calculation result of the resonator.

Schedule of the lecture

1. Introductions of the course, history, the relation of the electronic on the acoustics
2. Propagation of the sound in one dimensional system
3. Plane wave propagation and wave equation of the sound in three dimensional system
4. Acoustic impedance and acoustic absorption
5. Basic principle of the vibration
6. Analogies of acoustics to machineries and electronics
7. Kinds of an electro-acoustic transducer and an operation equation of the transducer
8. Design of principle of an electro-acoustic transducer (1)
9. Design of principle of an electro-acoustic transducer (2)
10. Hearing of sense
11. Digital signal processing for acoustics
12. Final exam and comments
13. Practice of sound filed calculation using FEM(1)
14. Practice of sound filed calculation using FEM(2)

Calculation of the Helmholtz resonator

Lecture slides for the derivation of the Helmholtz equation

Wave equation in the three dimensions

Equation of motion : $\rho_0 \frac{\partial \mathbf{v}}{\partial t} = -\nabla p = -\left(\frac{\partial p}{\partial x} \mathbf{i} + \frac{\partial p}{\partial y} \mathbf{j} + \frac{\partial p}{\partial z} \mathbf{k} \right)$

Conservation of mass: $\frac{\partial \rho}{\partial t} + \rho_0 \nabla \cdot \mathbf{v} = -\rho_0 \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \right)$

Gas state equation: $p = c^2 \rho$ (where $c^2 = \frac{\gamma p_0}{\rho_0}$)

Wave equation in the three dimensions

$$\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}$$

Helmholtz equation

In the acoustic phenomena, the time variation is expressed by the harmonic function $e^{j\omega t}$

Sound pressure $p = \rho_0 \frac{\partial \Phi}{\partial t} = j\omega \rho_0 \Phi$ Particle velocity $\mathbf{v} = -\nabla \Phi$

By substituting above equation to the wave equation, the following equation "Helmholtz equation" is obtained.

$$\nabla^2 \Phi + k^2 \Phi = 0$$

Demonstration of a Helmholtz resonator using can bottle

WaveSpectra
<http://efu.jp.net/soft/ws/ws.html>

Explanation of acoustic elements and a Helmholtz resonator

Acoustic element

Closed end tube: acoustic compliance

$$Z_a = \frac{p}{Q} = \frac{j\omega \rho_0 c}{S_a} \tan kL$$

Open end tube: acoustic mass (inertia)

$$Z_a = \frac{p}{Q} = j\omega \rho_0 L / S_a$$

Helmholtz resonator

Helmholtz resonator consists of small and short open end tube and short tank.

$$f = \frac{1}{2\pi} \sqrt{\frac{1}{C_a m_a}}$$

$$= \frac{c}{2\pi} \sqrt{\frac{S_a}{LV_a}}$$

Modeling of a can bottle and an analysis result

Finite element analysis: COMSOL

Analysis results (3D)

Finite element analysis: COMSOL

Analysis results (frequency - sound pressure)

Specification of the PC in the practice room
Windows 10, Core i5, Memory 25Gbyte

Lecture and Practice of the COMSOL by KESCO engineers

Before the practice of COMSOL, the students take two times lecture on FEM and COMSOL by KESCO engineers, Mr.Hashiguchi and Dr.Mi. After the lecture, students measurement by a bottle and analysis by COMSOL.

SHIBURA INSTITUTE OF TECHNOLOGY
2018.7.20/7.27
10:50-12:30

Pressure Acoustic Analysis with Finite Element Method (FEM)

Course 1: Introduction to COMSOL Multiphysics and basic modeling process

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What's COMSOL Multiphysics

COMSOL Multiphysics is a numerical simulation software which mainly using FEM method to solve various PDEs.

There are two features about COMSOL Multiphysics: ① Handling Multiphysics problems, ② Equation-based interface.

Multiphysics

Fluid / structure Electric / heat / structure Ray / heat / structure

Example of the slides on COMSOL lecture

Example of comments that the student wrote

* 6 Comments *

Experimental eigen freq. (blowing the bottle) *	473.7 Hz *
COMSOL eigen freq. *	482.38 Hz *
Calculated eigen freq (Helmholtz resonator) *	434.46 Hz *

The eigen frequencies obtained from the COMSOL and from the formula from the Helmholtz resonator differ from the experimental one most likely due to the approximations of the measures taken from the bottle. *

The value from the Helmholtz resonator is specially influenced by the calculated values from the volume and the length from the bottle. The frequency obtained is probably farther than the actual value, when compared to the one obtained with the Finite Element Method through COMSOL. *

Practice and subjects of the final report (1)

- 1) Select a bottle among 4 kinds prepared ones. And measure the dimensions of the bottle.
- 2) Estimate roughly eigenfrequency of a bottle using the concept of Helmholtz resonator.
- 3) Make a sound by blowing the bottle. And check the frequency using the spectrum analysis application on the notebook PC.

Practice and subjects of the final report (2)

- 4) Compare the measured frequency with the rough estimation result.
- 5) Simulate both eigenfrequency and frequency domain analysis using COMSOL FEM application.
- 6) Compare the FEM analysis result with the measured frequency.

Practice and subjects of the final report (3)

- 7) Create the report using COMSOL function.
- 8) Add your comment on the rough estimation result and the FEM analysis one with the report file COMSOL created.
- 9) Submit the report to the Submission folder in the ShareFolder. The file name should be changed Student_ID.docx.

Subjects of the COMSOL practice

Summaries

Students created their subject report by COMSOL report function. They also have to add their comment on comparison between the measurement, rough estimation and FEM results. Our future plan is to make the students consider the results more deeply.