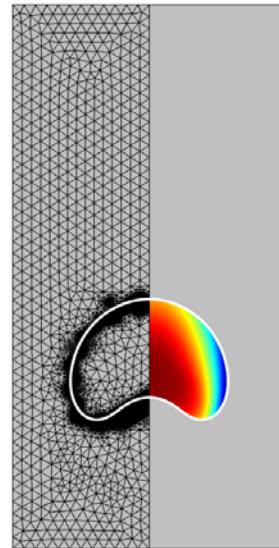


# Adaptive mesh refinement: Quantitative computation of a rising bubble using COMSOL Multiphysics®

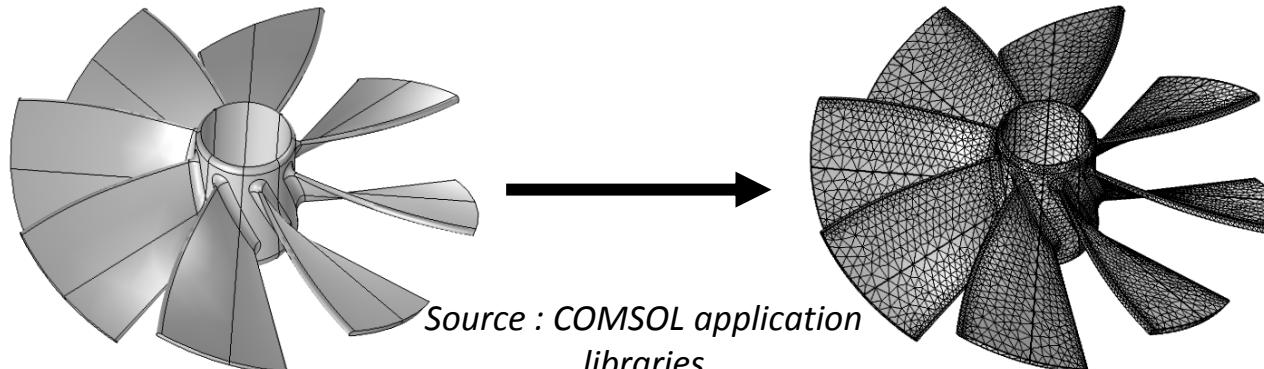


T.Preney, J.D. Wheeler and P.Namy  
SIMTEC- (+33) 9 53 51 45 60  
[patrick.namy@simtecsolution.fr](mailto:patrick.namy@simtecsolution.fr)

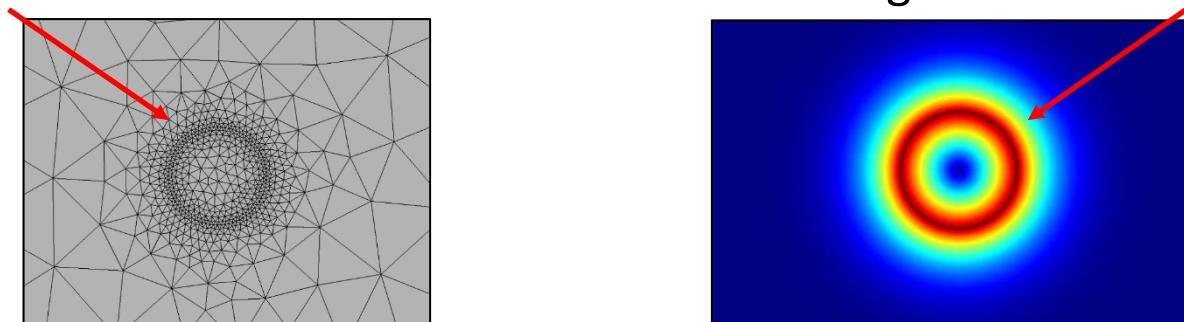
**COMSOL**  
**CONFERENCE**  
2018 BOSTON

# Need to accelerate your calculations ?

- The mesh : fundamental pillar of numerical computation on which the approached solution is built



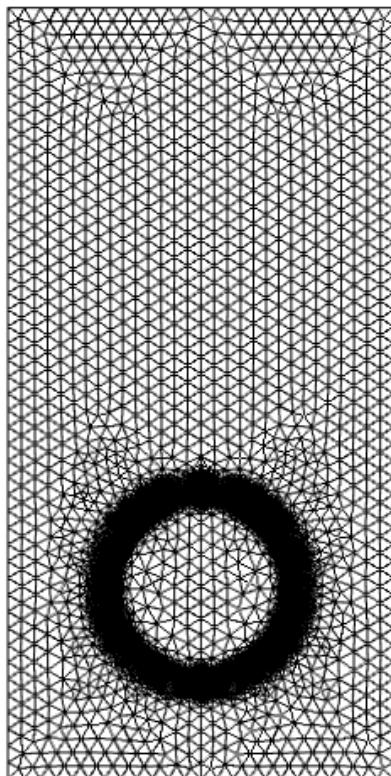
- A high concentration of nodes is needed where the gradients are important



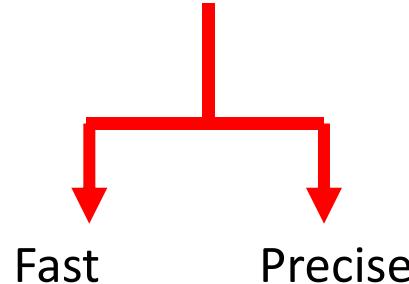
→ May induce large computational times !

# Introduction

Definition : To adapt the mesh to the solution as time goes by



→ More efficient computation



# Working with SIMTEC

## Industry Challenges

- R&D sections: experts in their field  
→ Expertise in numerical modelling?
- Lack of time
- FE modelling performed by a small group of people



## SIMTEC's Solutions

- Numerical modelling project
  - SIMTEC's member as your colleague
  - Help improve your modelling knowledge!
  - Cost-effective outsourcing



## Our team & Our clients

6 members all EngD + PhD

- Extensive research background
- Complex problems / various fields of expertise

Successful track record:

- Big companies
- Government laboratories

Involved in research consortia

- EU funded projects (REEcover / SHARK)
- PhD projects supervision.



Numerical modelling /  
simulation consultants

 COMSOL  
*Certified Consultant*



[www.simtecsolution.fr/en](http://www.simtecsolution.fr/en)

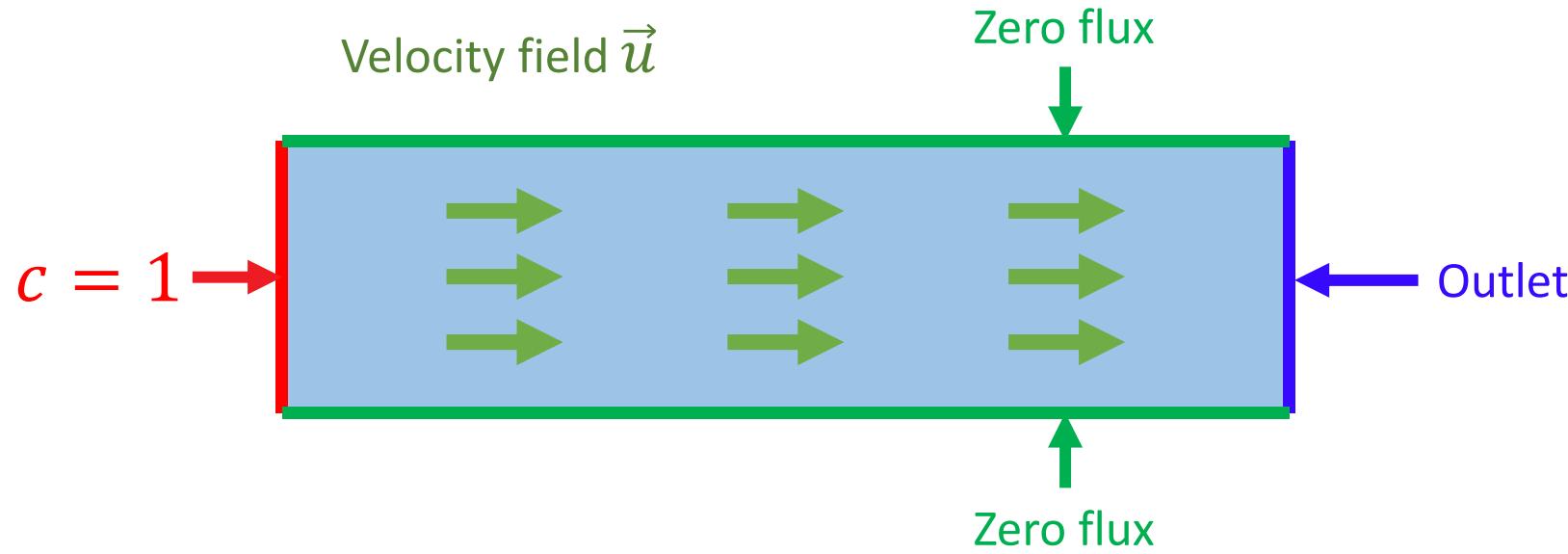
# Plan

- I. General principle
- II. 2D validation study
- III. 3D validation study : comparison with other softwares

# Introduction

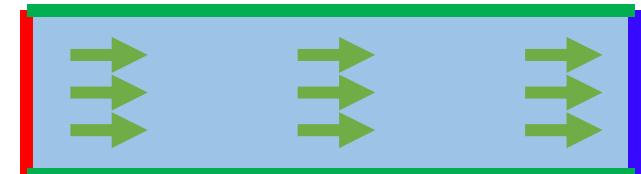
Especially useful for a time-dependent study !

Example: transport of a concentration in water

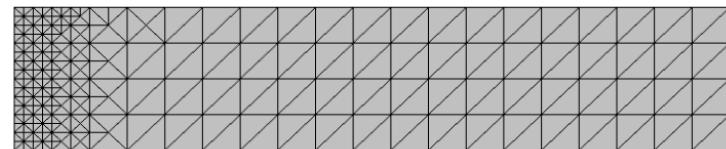
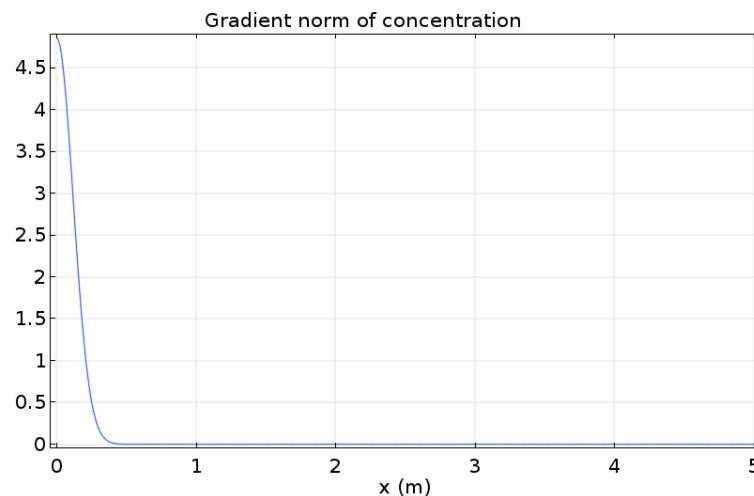


→ Concentration front propagation

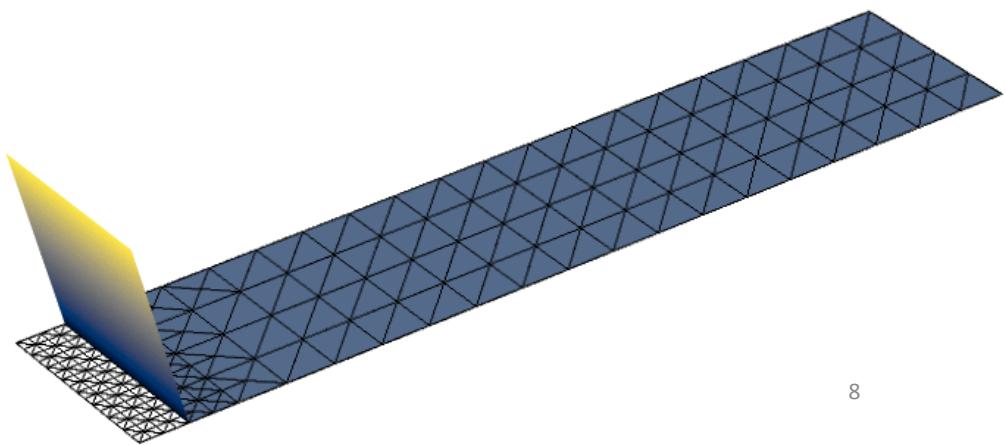
# Introduction



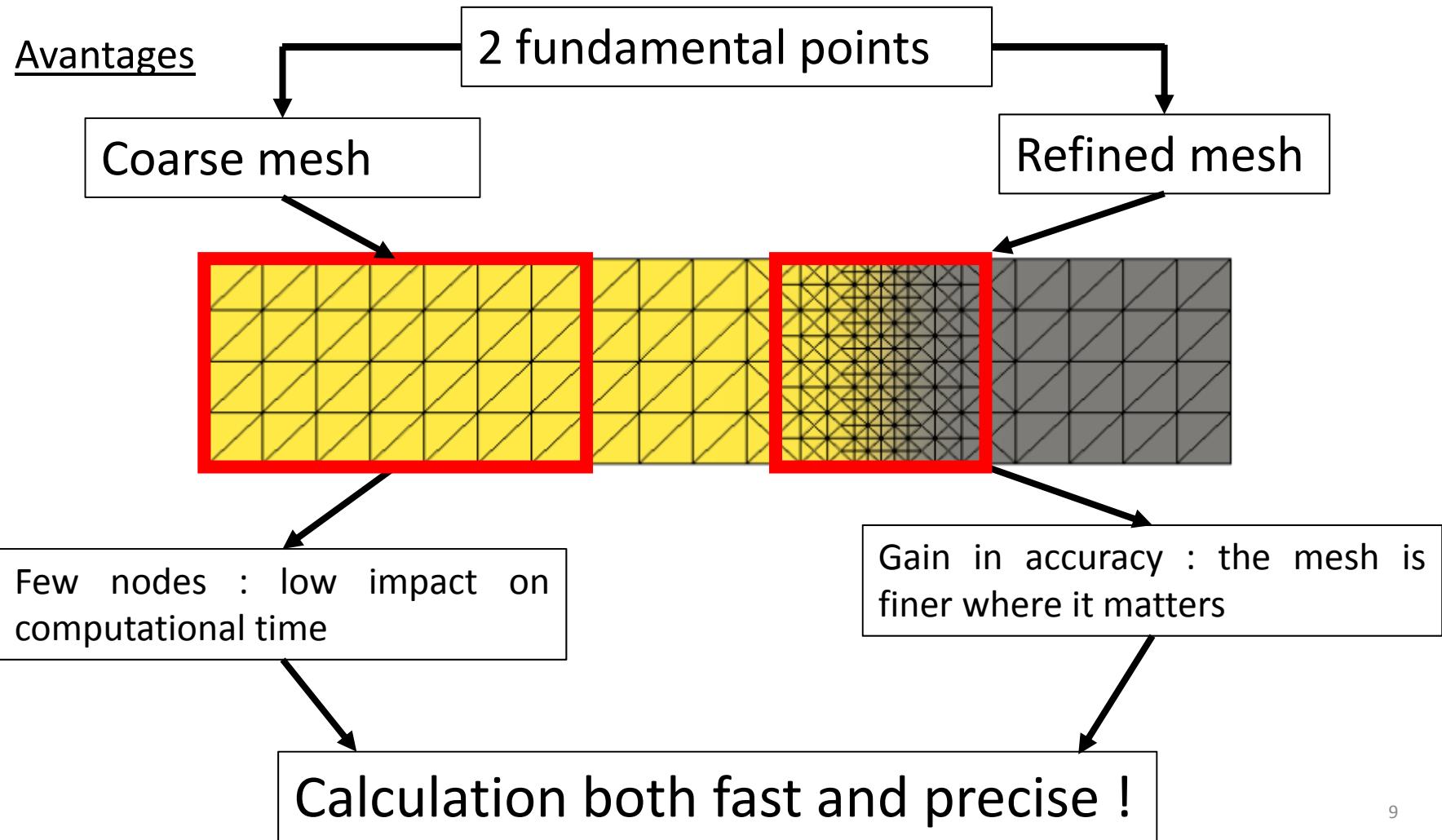
Idea : Refine the mesh where the concentration gradient norm is important



About the concentration:

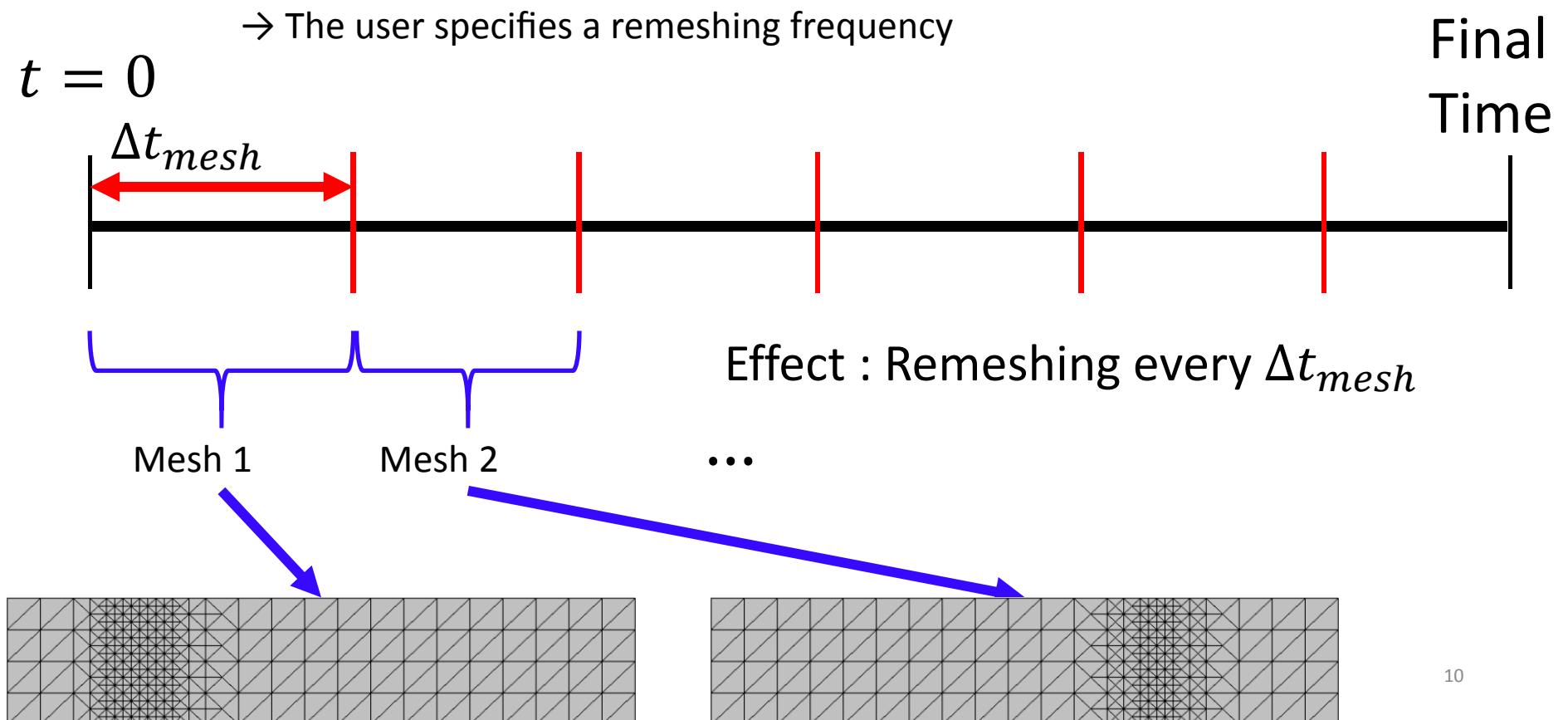


# I – General principle



# I – General principle

Question : How will the mesh evolve?

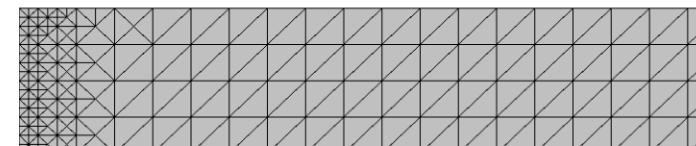
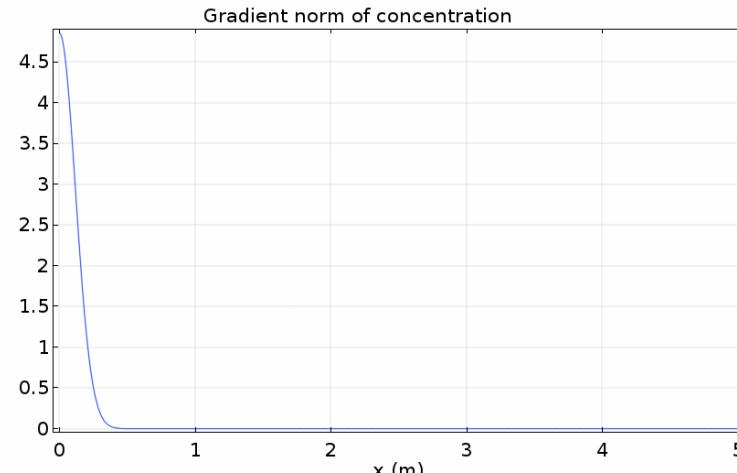


# I – General principle

Question : Where will the mesh be adapted?

→ The user specifies an error indicator (usually a gradient norm)

Effect : Mesh refinement where  
the error indicator function is  
important



# I – General principle

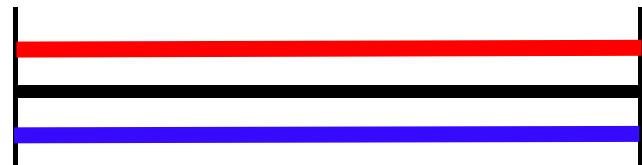


1. First calculation : estimation of the error indicator on the coarse mesh  
To determine spatial areas where the indicator is important
2. Mesh refinement on those areas
3. Second calculation : computation of the solution on the (now) refined mesh
4. Back to step 1

# I – General principle

Double calculations sweep

$$t + \Delta t \qquad t + 2\Delta t$$

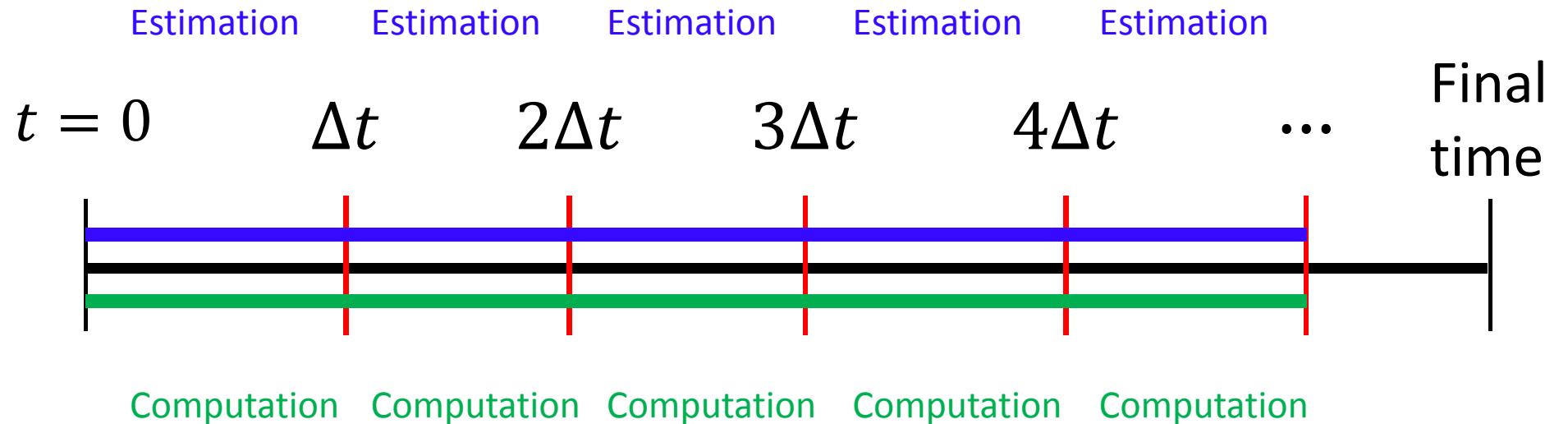


- 1. First calculation : estimation of the error indicator on the coarse mesh
- 2. Mesh refinement where the error indicator is important
- 3. Second calculation : computation of the solution on the refined mesh
- 4. Back to step 1 at the end of the time interval

# I – General principle

Double calculations sweep

1. Estimation : low precision calculation on coarse mesh
2. Mesh adaptation
3. Precise calculation on refined mesh



## II – 2D Study

Public benchmark available at

<http://www.featflow.de/en/benchmarks/cfdbenchmarking/bubble.html>

Reference paper:

Hysing, S.; Turek, S.; Kuzmin, D.; Parolini, N.; Burman, E.; Ganesan, S.;  
Tobiska, L.: Quantitative benchmark computations of two-dimensional  
bubble dynamics, International Journal for Numerical Methods in Fluids,  
Volume 60 Issue 11, Pages 1259-1288, DOI: 10.1002/fld.1934, 2009

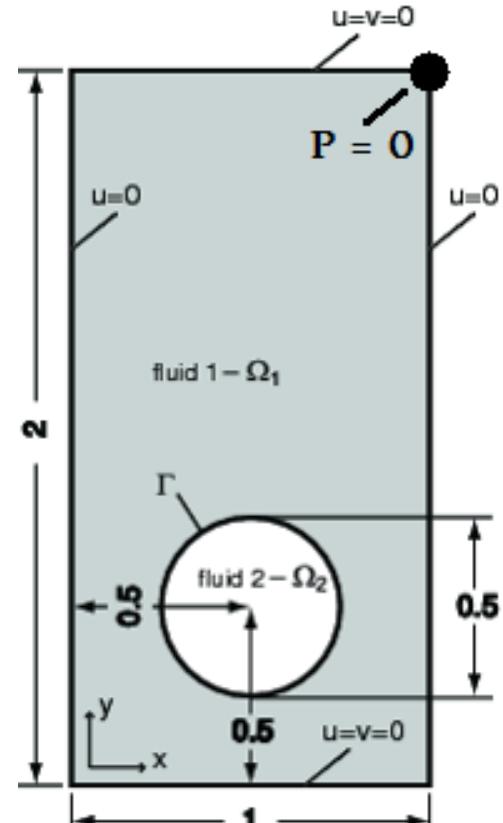
## II – 2D Study

### Configuration

Rise of a bubble of gas inside a fluid

- 2D geometry
- Laminar flow modelled by Navier-Stokes equations
- Two-phase flow with a phase-field approach

Study parameters



Extract : reference paper

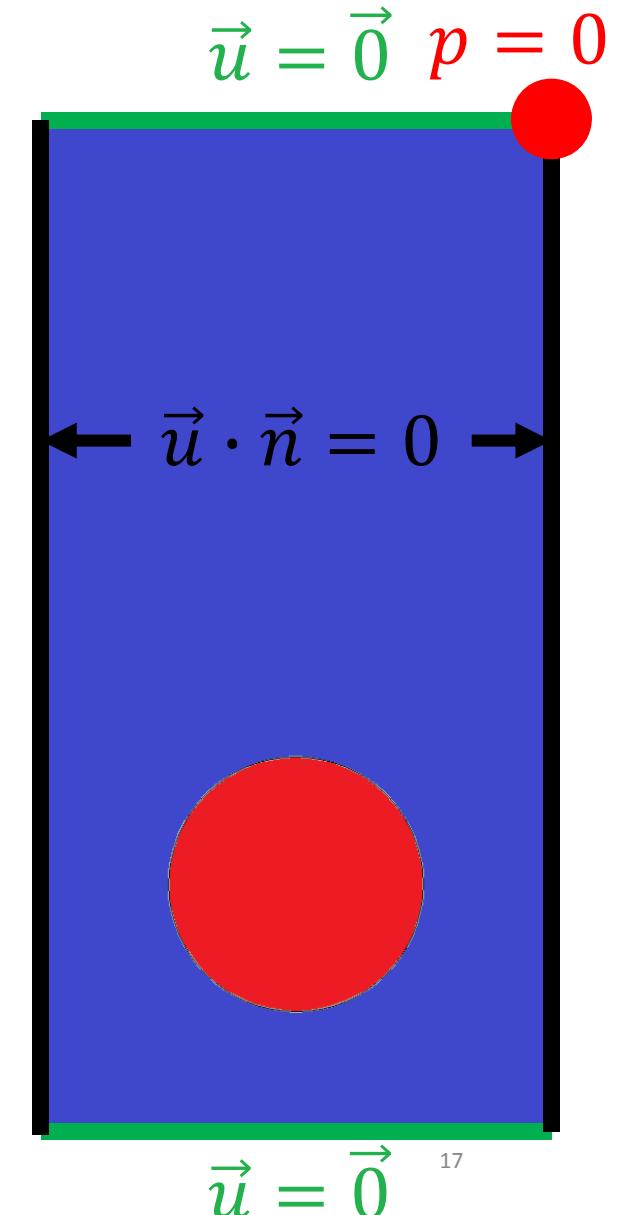
$\rho_1$ ( $kg \cdot m^{-3}$ )	$\rho_2$ ( $kg \cdot m^{-3}$ )	$\mu_1$ ( $Pa \cdot s$ )	$\mu_2$ ( $Pa \cdot s$ )	$g$ ( $m \cdot s^{-2}$ )	$\sigma$ ( $N \cdot m^{-1}$ )
1000	1	10	0,1	0,98	1,96

## II – 2D Study

Equations and boundary conditions

Laminar flow with Navier-Stokes equations

$$\begin{cases} \rho \left( \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right) = \rho \vec{g} - \vec{\nabla} p + \mu \Delta \vec{u} \\ \operatorname{div}(\vec{u}) = 0 \end{cases}$$

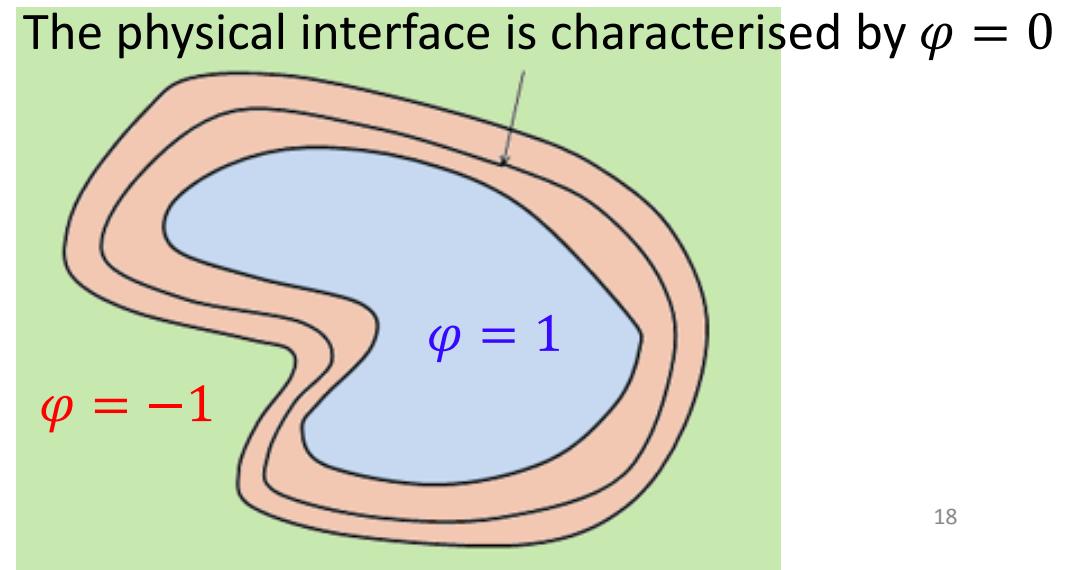


## II – 2D Study

### Phase-field method to simulate two-phase flow

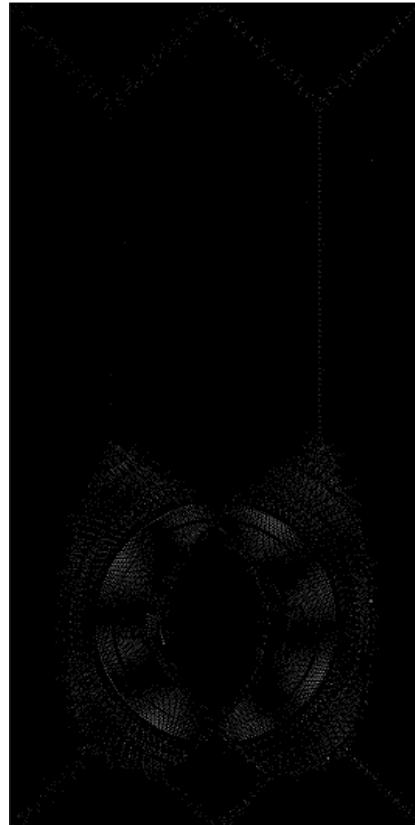
Principle : Use a dimensionless phase field variable  $\varphi$  that can take values in  $\{-1, 1\}$  according to the phase represented

- Fluid 1 :  $\varphi = -1$
- Fluid 2 :  $\varphi = 1$

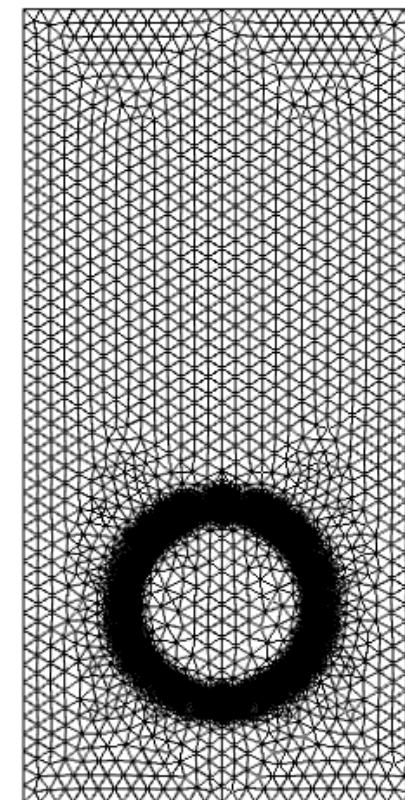


## II – 2D Study

Two test cases: fixed mesh and adaptive mesh



Fine mesh



Adaptive mesh

## II – 2D Study

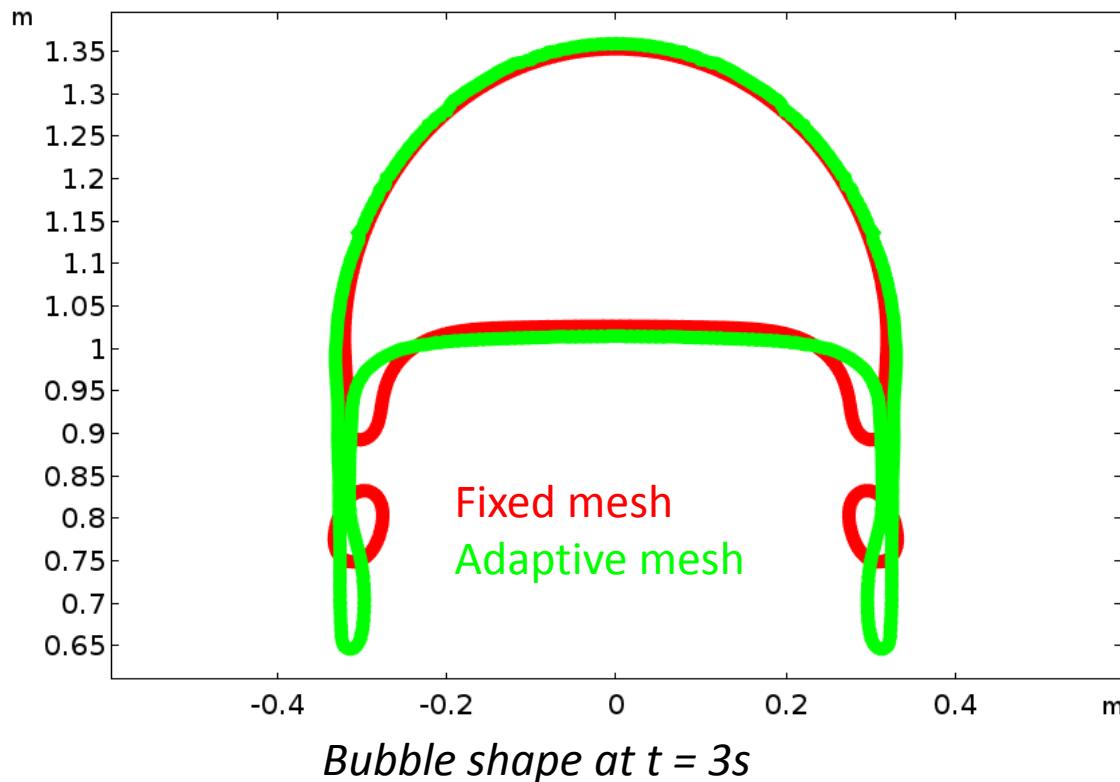
Mesh type	Mesh element size	Number of degrees of freedom	Computational time
Fixed	$6,4 * 10^{-3} \text{ m}$	260 000	75 minutes
Adaptive	$5,4 * 10^{-3} \text{ m}$	250 000	15 minutes ↗/5 !

→ Massive acceleration !

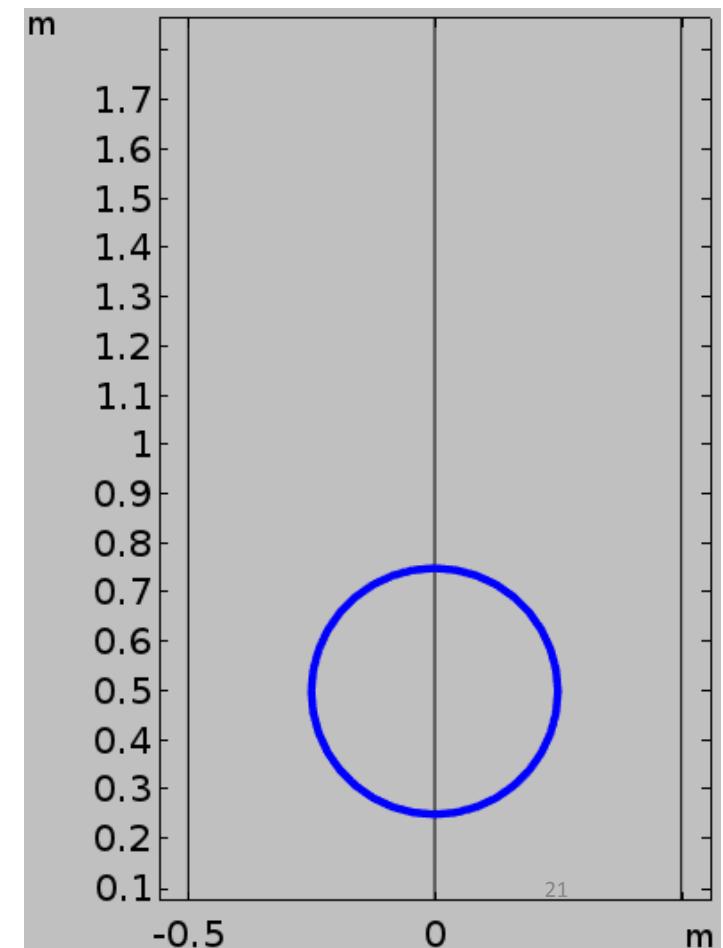
What about accuracy?

## II – 2D Study

Results : bubble shape and comparison



→ Good adequacy generally ...  
... but some details vary (satellites)

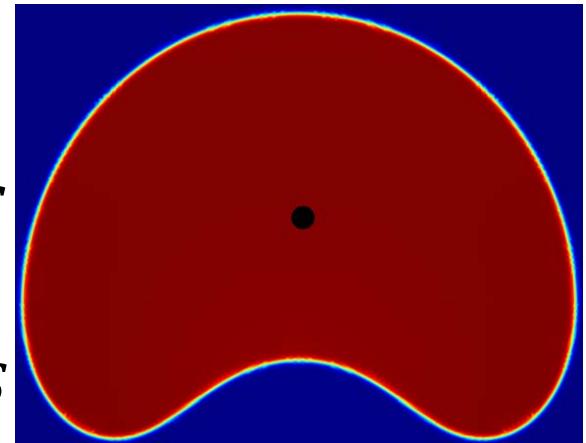


## II – 2D Study

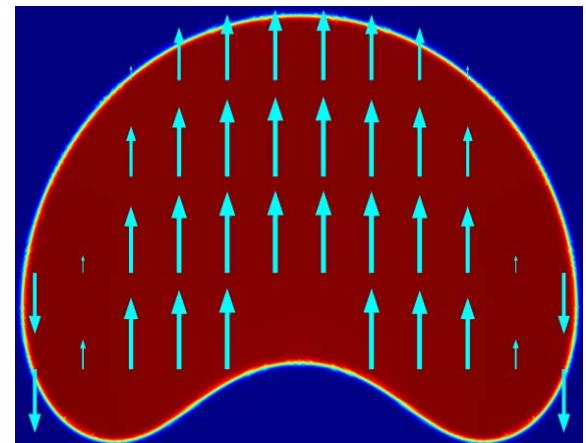
### Quantitative comparison criteria

1. Position of centre of mass of the bubble  $\bar{y} = \frac{1}{|\Omega|} \int_{\Omega} y \, dS$

where  $\Omega = \{X \in \mathbb{R}^2 \mid \varphi(X) \geq 0\}$  and  $|\Omega| = \int_{\Omega} dS$

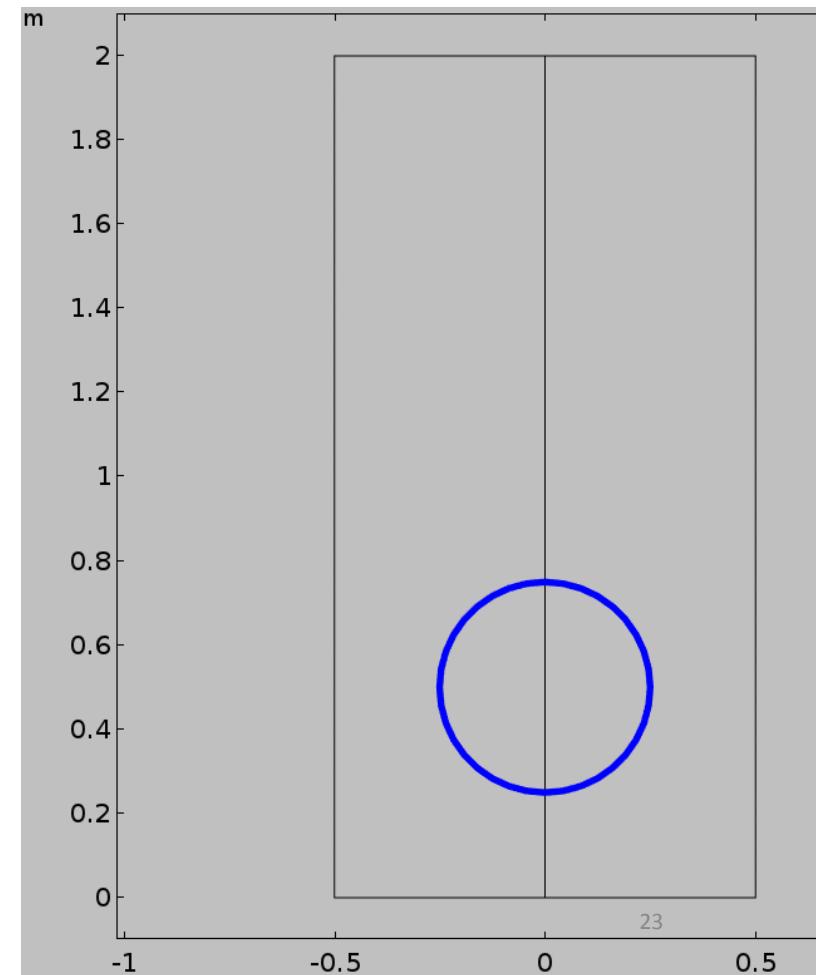
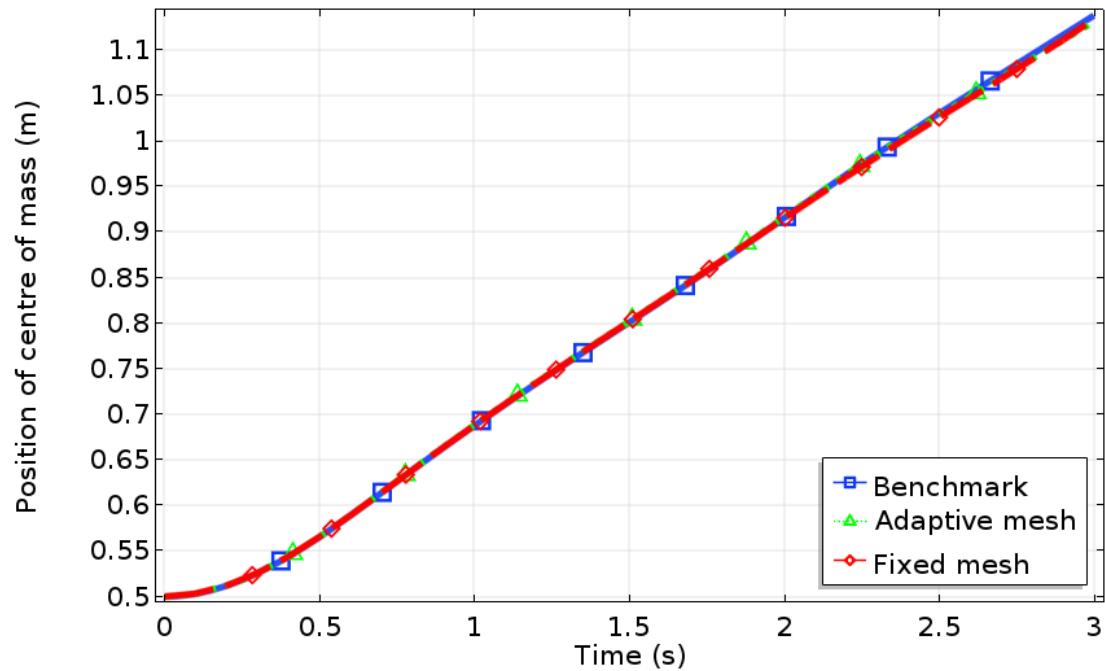


2. Mean rise velocity  $\bar{v} = \frac{1}{|\Omega|} \int_{\Omega} v \, dS$



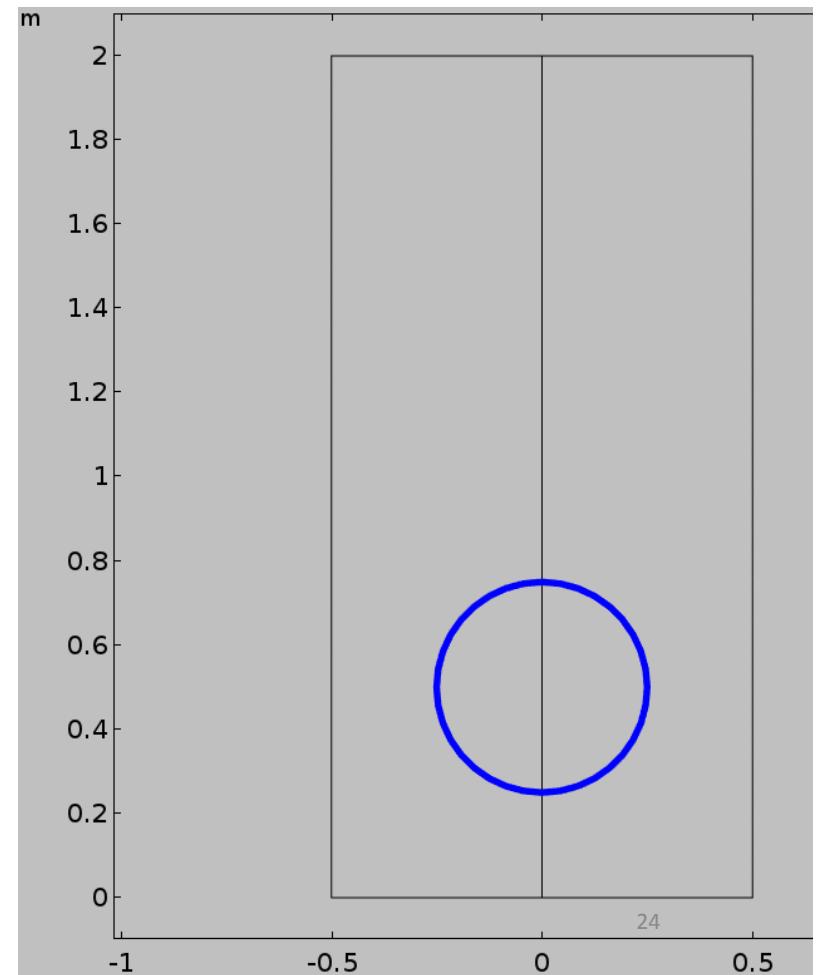
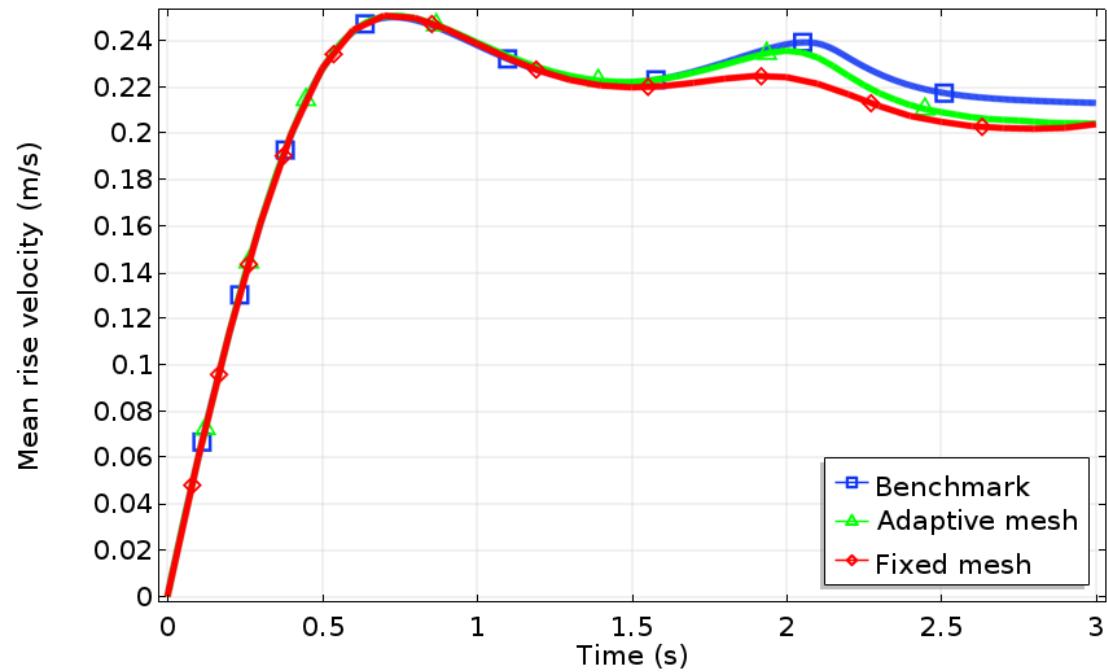
## II – 2D Study

Results: comparison with the benchmark



## II – 2D Study

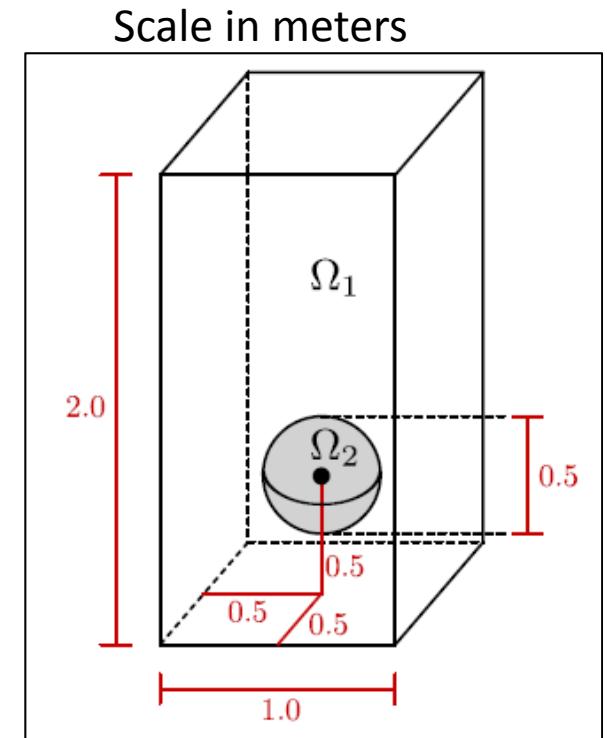
Results: comparison with the benchmark



## III – 3D study

### Configuration

3D generalisation of the 2D case



### **From the reference paper**

J. Adelsberger, P. Esser, M. Griebel, S. Groß, M. Klitz, and A. Rüttgers.

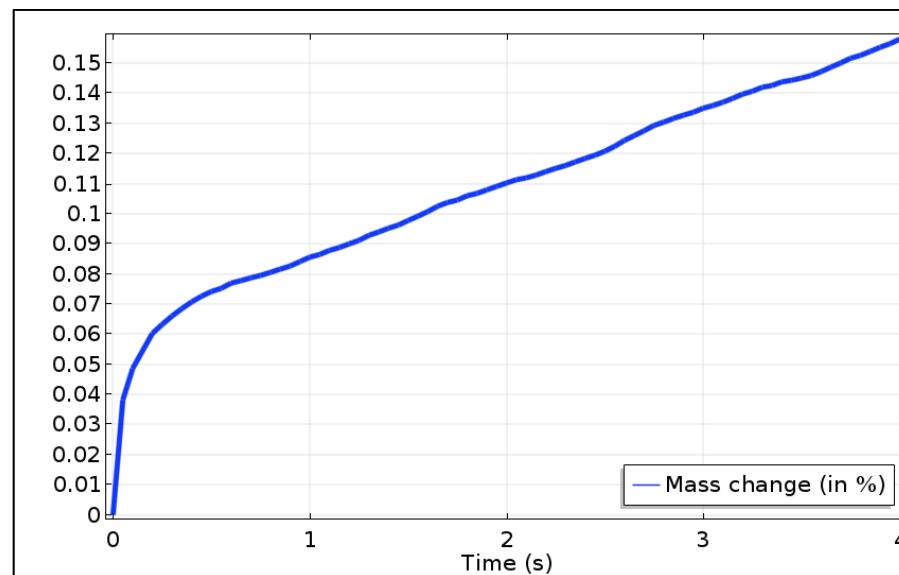
### 3D incompressible two-phase flow benchmark computations for rising droplets.

2014. Proceedings of the 11th World Congress on Computational Mechanics (WCCM XI), Barcelona, Spain, also available as INS Preprint No. 1401 and as IGPM Preprint No. 393.

*Extract : reference article*

## III – 3D study

### Numerical validation



Total mass variation < 0,2%

## III – 3D study

Comparison with two CFD software

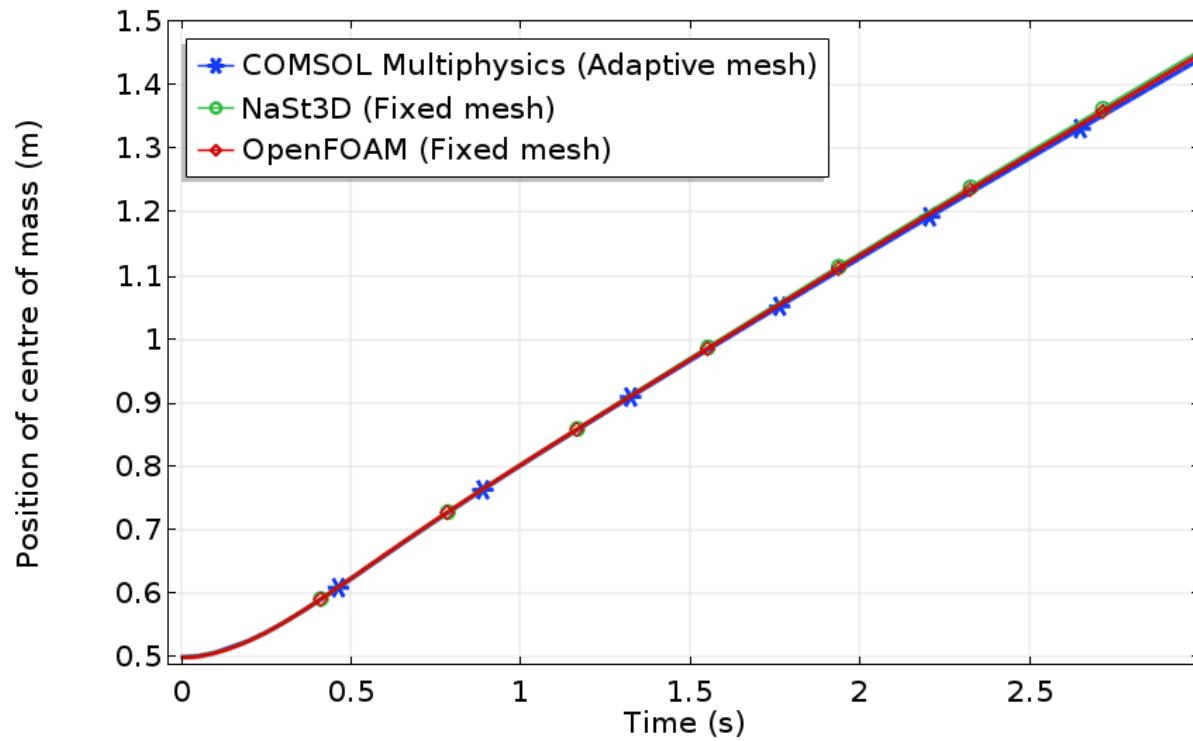
- NaSt3D
- OpenFOAM

### Computational times

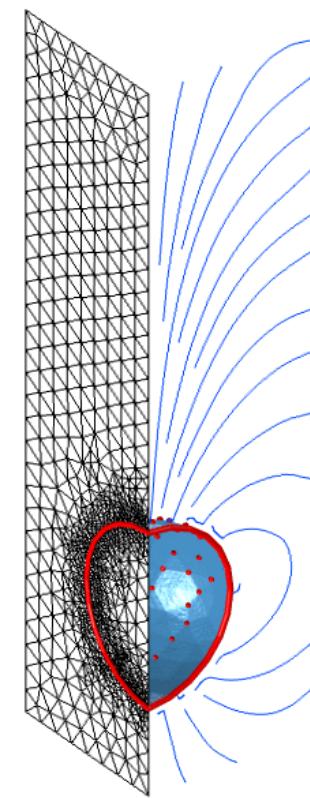
COMSOL Multiphysics® (adaptive mesh)	22 h on 2 cores at 4,16 GHz
NaSt3D (maillage fixe)	1 week on 32 cores at 2,226 GHz
OpenFOAM (maillage fixe)	2,5 days on 32 cores at 2,226 GHz

## III – 3D study

### Results: comparison with the benchmark

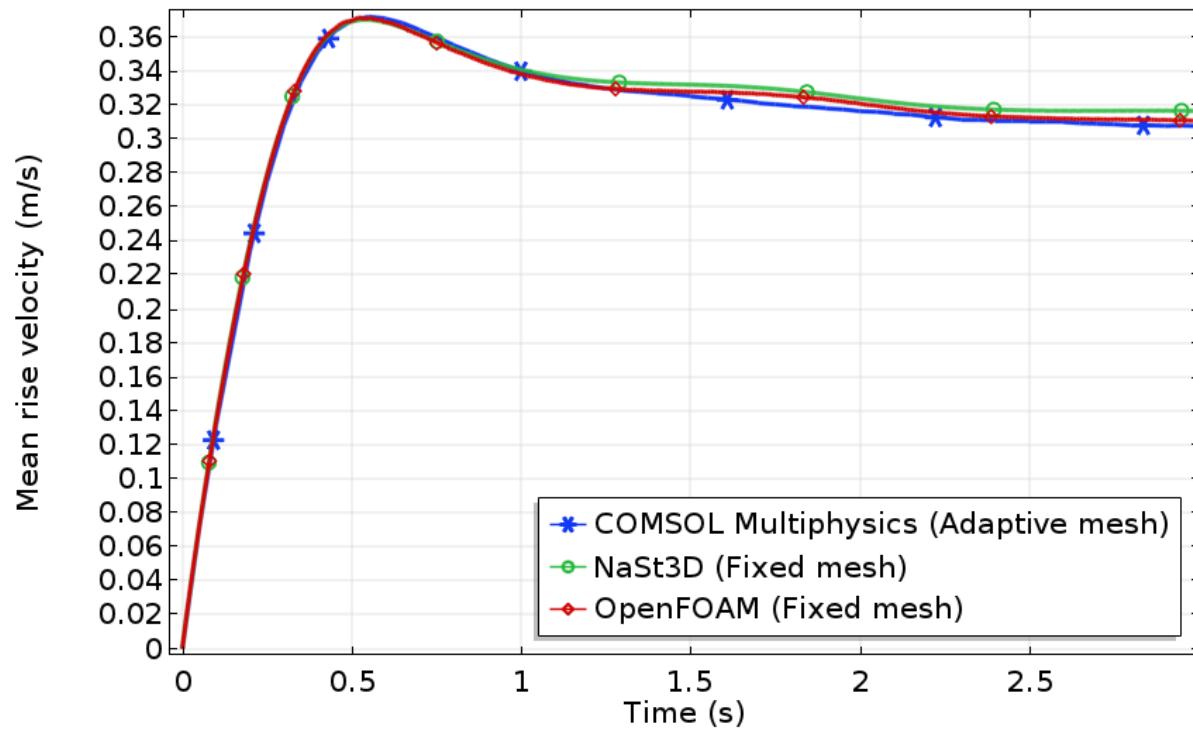


*Mesh visualisation (left)  
streamlines (right)  
interface (in red)*

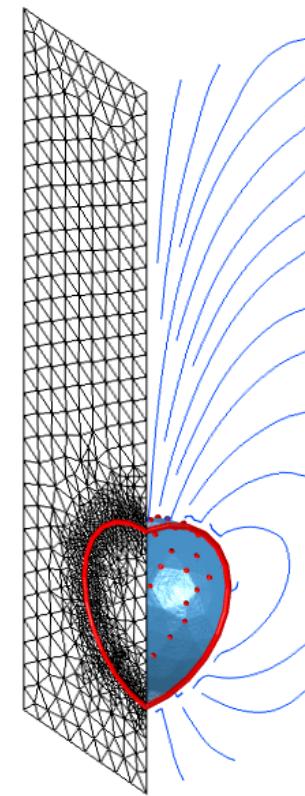


## III – 3D study

Results: comparison with the benchmark



*Mesh visualisation (left)  
streamlines (right)  
interface (in red)*

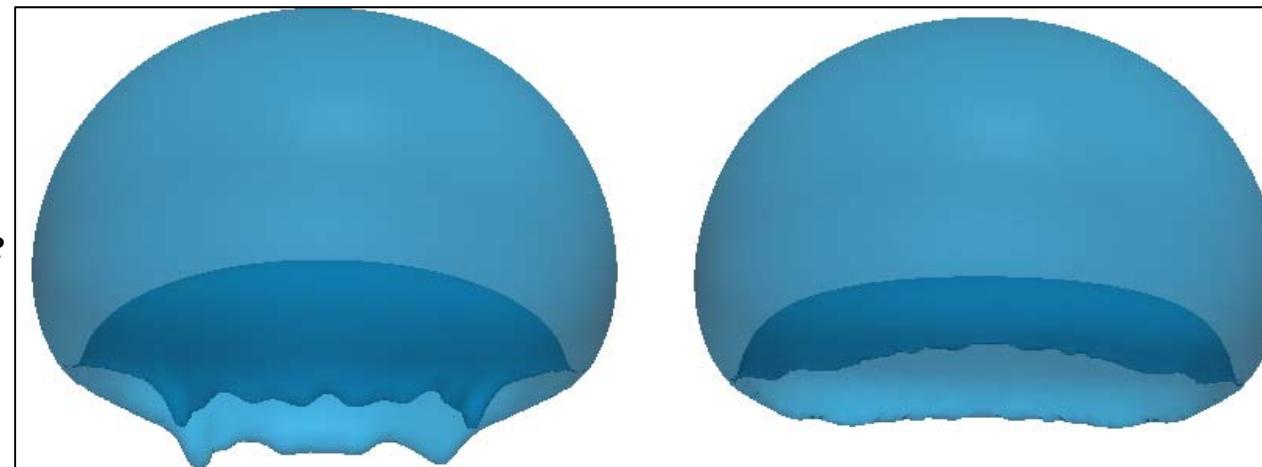


## III – 3D study

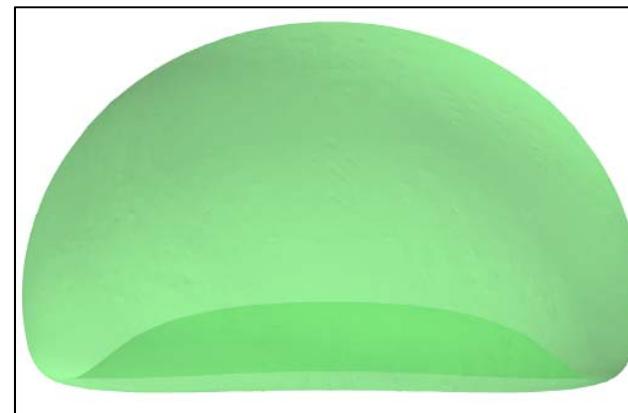
Results: comparison of the bubble shape at t=3,5s

*Extract:  
reference article*

NaSt3D

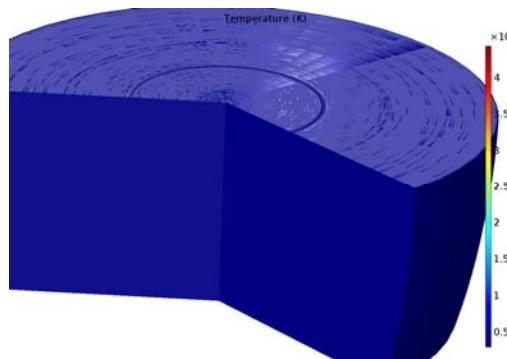


OpenFOAM

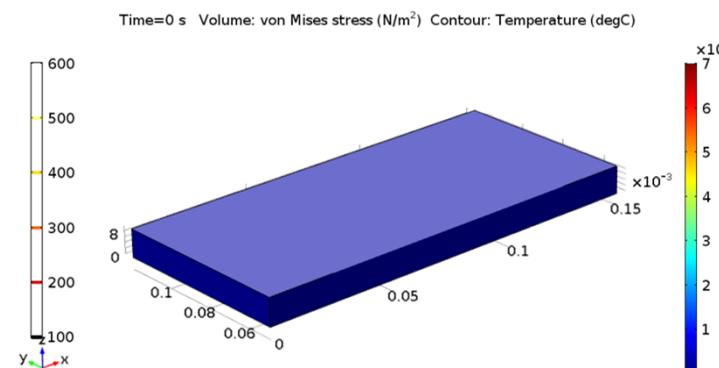


# Conclusion

- Principle of the adaptive mesh refinement method : accelerate calculations while improving accuracy
- Comparison with results from literature and others CFD software : **validation of the method**
- Practical applications on industry topics:



*Laser piercing*



*Additive fabrication*

# Thanks a lot!

## Q&A



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