UNIVERSITY of WASHINGTON

Presented at the COMSOL Conference 2010 Boston

Dimensionless versus Dimensional Analysis in CFD and Heat Transfer

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October 2010

Overview

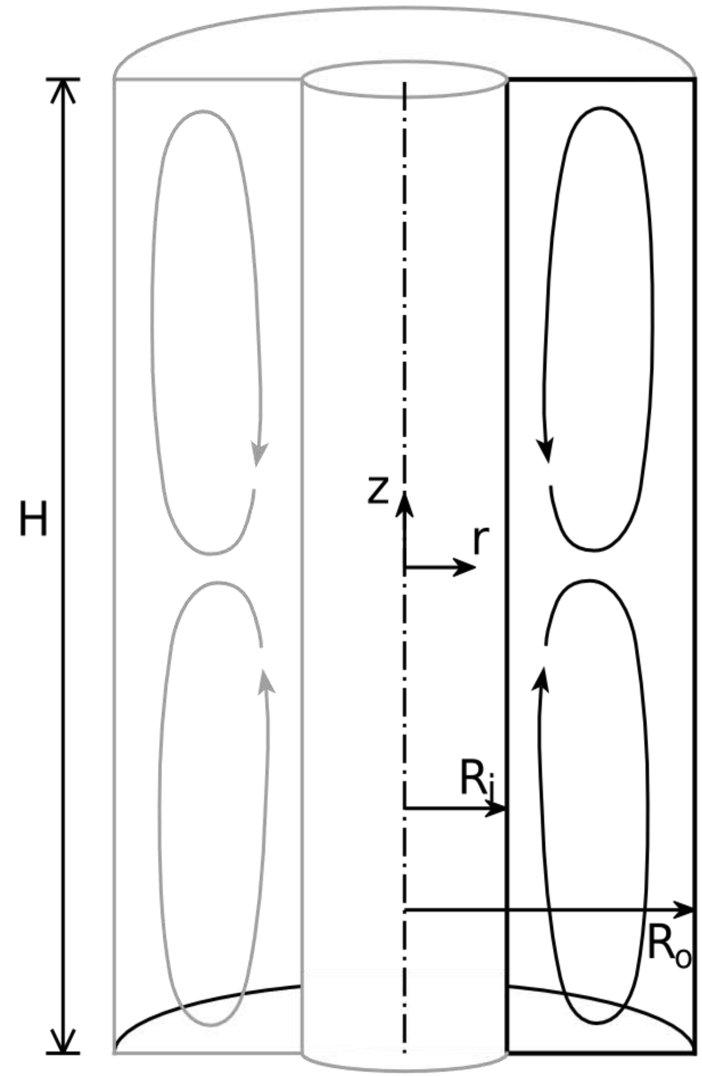
- Problem Introduction
- Literature Review
- Model (Governing) Equations
- Simulation Results
- Dimensioned and Dimensionless Results
- Conclusions

Problem Introduction

- There are many ways of non dimensionalizing natural convection flow problems and it is not clear which is best.
- The specific geometry and problem provide a platform to test different methods in COMSOL.

$$A = \frac{H}{R_o - R_i}$$

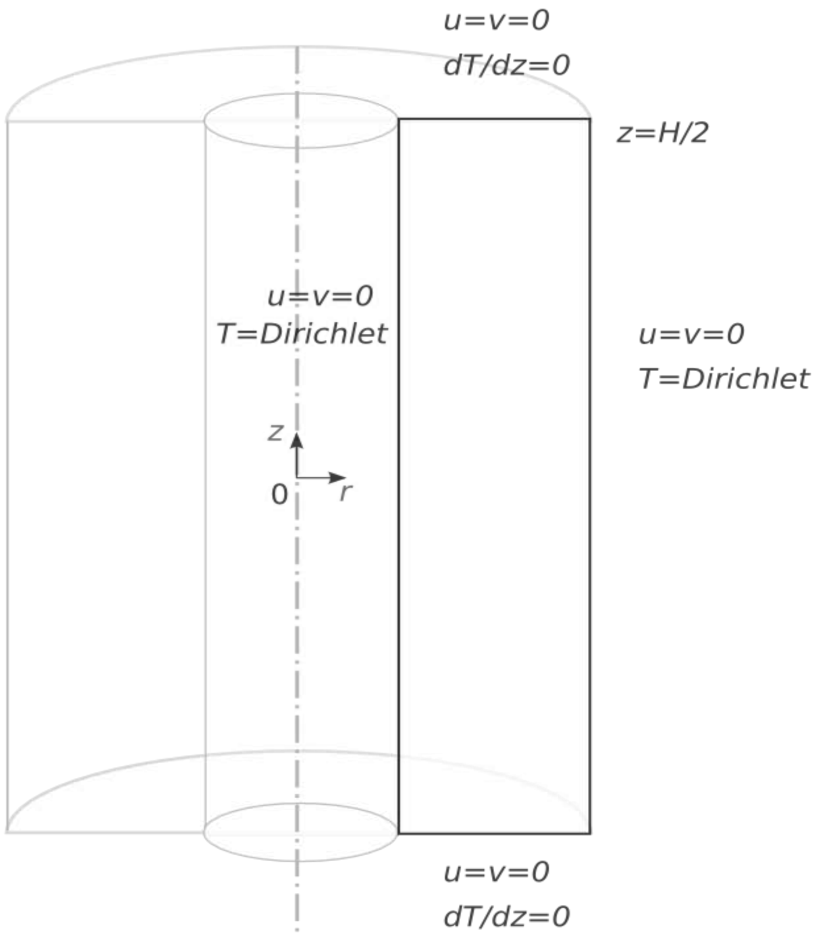
$$\eta = \frac{Ro}{R_i}$$

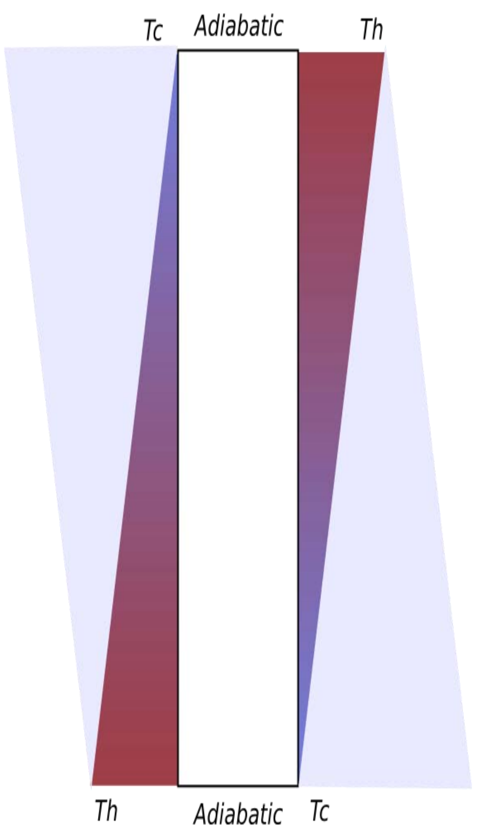


Literature Review

Author	Year	A	Description
De Vahl Davis [5]	1983	1	Benchmark solution for a square cavity.
Lee and Korpela [9]	1983	0-1000	Reported Nu and streamfunctions.
Chenoweth and Paolucci [2]	1986	1-10	Compare ideal gas and Boussinesq.
Suslov and Paolucci [17]	1995	∞	Non-Boussinesq impact on stability and considered Ra_c with ΔT.
Mlaouah et al. [11]	1997	1	Compared Boussinesq, ideal gas, and low Mach approximation.
Paillere et a. [13]	2000	1	Compared Boussinesq and low Mach approximation.
Xin and Le Quere [20]	2002	8	Benchmark study reported Ra_c.
Christon et al. [3]	2002	8	Comparison study of methods, grids, etc.
Reeve et al. [16]	2003	10	Commercial code FIDAP.
Vierendeels et al. [19]	2003	1	Benchmark with ideal gas.
Xin and Le Quere [21]	2006	1-7	Investigated instabilities.
Dillon et al. [6]	2009	8-33	Dimensioned benchmark study in rectangular cavity, COMSOL.

Problem Introduction





Model Equations

Navier Stokes

$$\rho\frac{\partial u}{\partial t} + \rho(u \cdot \nabla)u = \nabla \cdot (-pI + \eta(\nabla u + \nabla u^T)) - (2\eta/3 \nabla \cdot u)I + F \quad (5)$$

Conservation of Mass

$$\nabla \cdot u = 0 \quad (6)$$

Conservation of Energy

$$\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot (-k \nabla T) = -\rho c_p u \cdot \nabla T \quad (7)$$

Characteristic Velocity Options

Author	Characteristic Velocity	Description
	$\frac{\alpha}{L} = \frac{k}{\rho c_p L}$	Thermal diffusion velocity.
De Vahl Davis [5]	$\frac{\sqrt{\beta g \Delta T L}}{\sqrt{Gr}} = \frac{\mu}{\rho L}$	Viscous diffusion velocity.
Ostrach [12]	$\sqrt{\beta g \Delta T L}$	For strongly coupled flows $Pr < 1$ and $\sqrt{Gr} > 1$
Ostrach [12]	$\frac{\sqrt{\beta g \Delta T L}}{\sqrt{Pr}}$	For strongly coupled flows $Pr > 1$ and $\sqrt{Gr} > 1$
Wan Hassan [8]	$\frac{\alpha Ra^{1/4}}{L} = \frac{k Ra^{1/4}}{\rho c_p L}$	Based on boundary layer thickness and thermal diffusion velocity.
Abrous [1]	$\frac{\mu Ra^{1/4}}{\rho L}$	Based on boundary layer thickness and viscous diffusion velocity.

Dimensionless Variables

Parameter	Option 1	Option 2 Strongly coupled	Option 3 Weakly coupled
R, Z	$\frac{r}{H}, \frac{z}{H}$	$\frac{r}{H}, \frac{z}{H}$	$\frac{r}{H}, \frac{z}{H}$
U	$\frac{u}{\frac{\alpha}{L}\sqrt{RaPr}}$	$\frac{u}{\sqrt{g\beta\Delta T H}}$	$\frac{u}{u_{forced}}$
V	$\frac{v}{\frac{\alpha}{L}\sqrt{RaPr}}$	$\frac{v}{\sqrt{g\beta\Delta T H}}$	$\frac{v}{u_{forced}}$
Θ	$\frac{T-T_c}{\Delta T}$	$\frac{T-T_c}{\Delta T}$	$\frac{T-T_c}{\Delta T}$
τ	$t\sqrt{g\beta\Delta T H^{-1}}$		
P	$\frac{pL}{\mu\frac{\alpha}{L}\sqrt{RaPr}}$		
ρ	$\sqrt{\frac{Ra}{Pr}}$	1	1
c_p	Pr	1	1
μ	1	$\sqrt{\frac{Pr}{Ra}}$	$\frac{1}{Re}$
g	1		
β	1	1	NA
k	1	$\frac{1}{\sqrt{RaPr}}$	$\frac{1}{Pe}$
F	$(T-T_c)\sqrt{\frac{Ra}{Pr}}$	$\sqrt{\frac{Ra}{Pr}}$	Re

Dimensioned Variables

Description	**Equation**
Boussinesq Approximation	$\rho = \rho_o(1 - \beta(T - T_c))$
Ideal Gas	$\rho = P/RT$
Force Term	$F = -g\rho$

Simulation Results

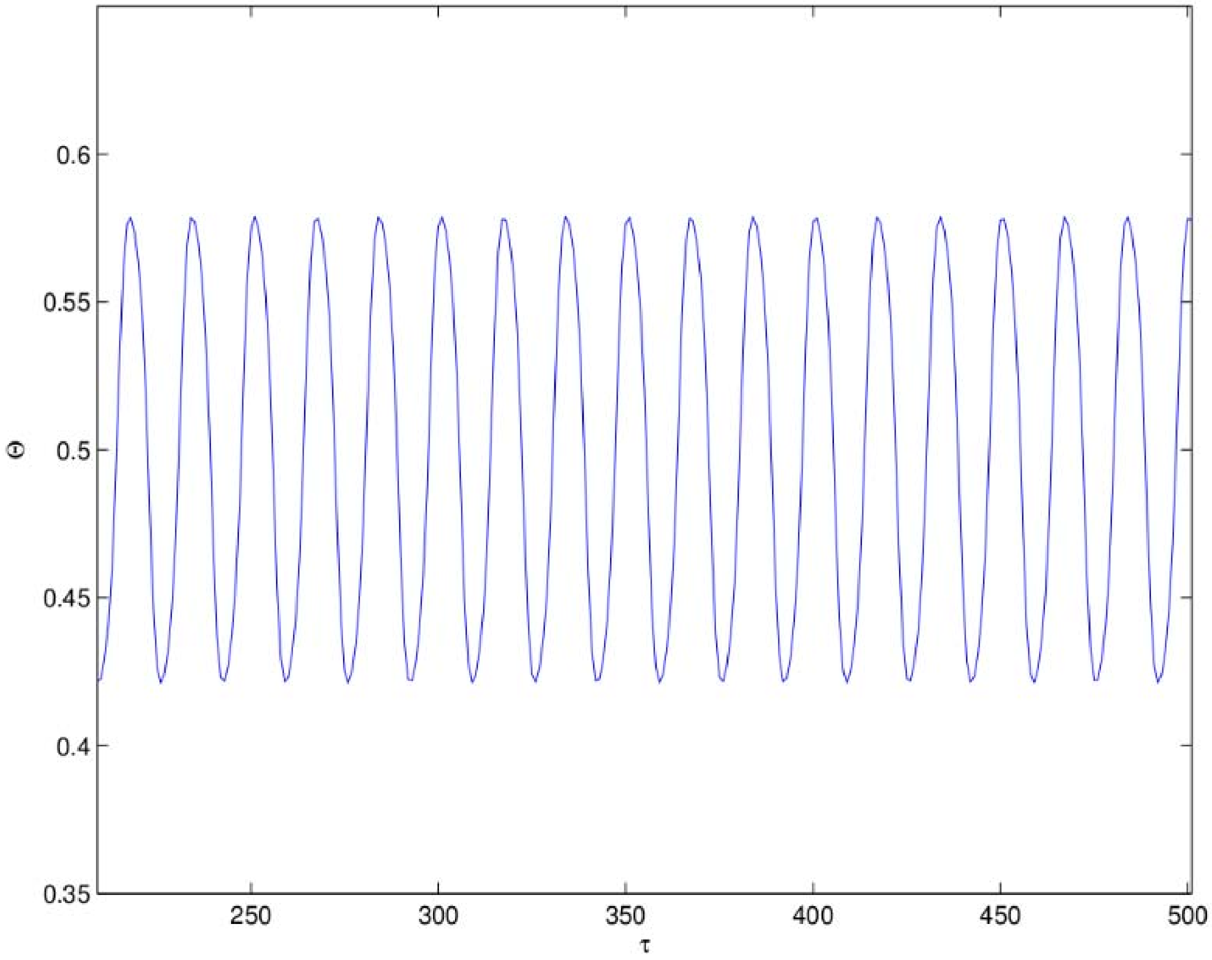


Figure 1: Dimensionless temperature at the center of the cavity over time. $Ra = 2.5e7$, $A = 10$ and $\eta = 0.6$.

Simulation Results

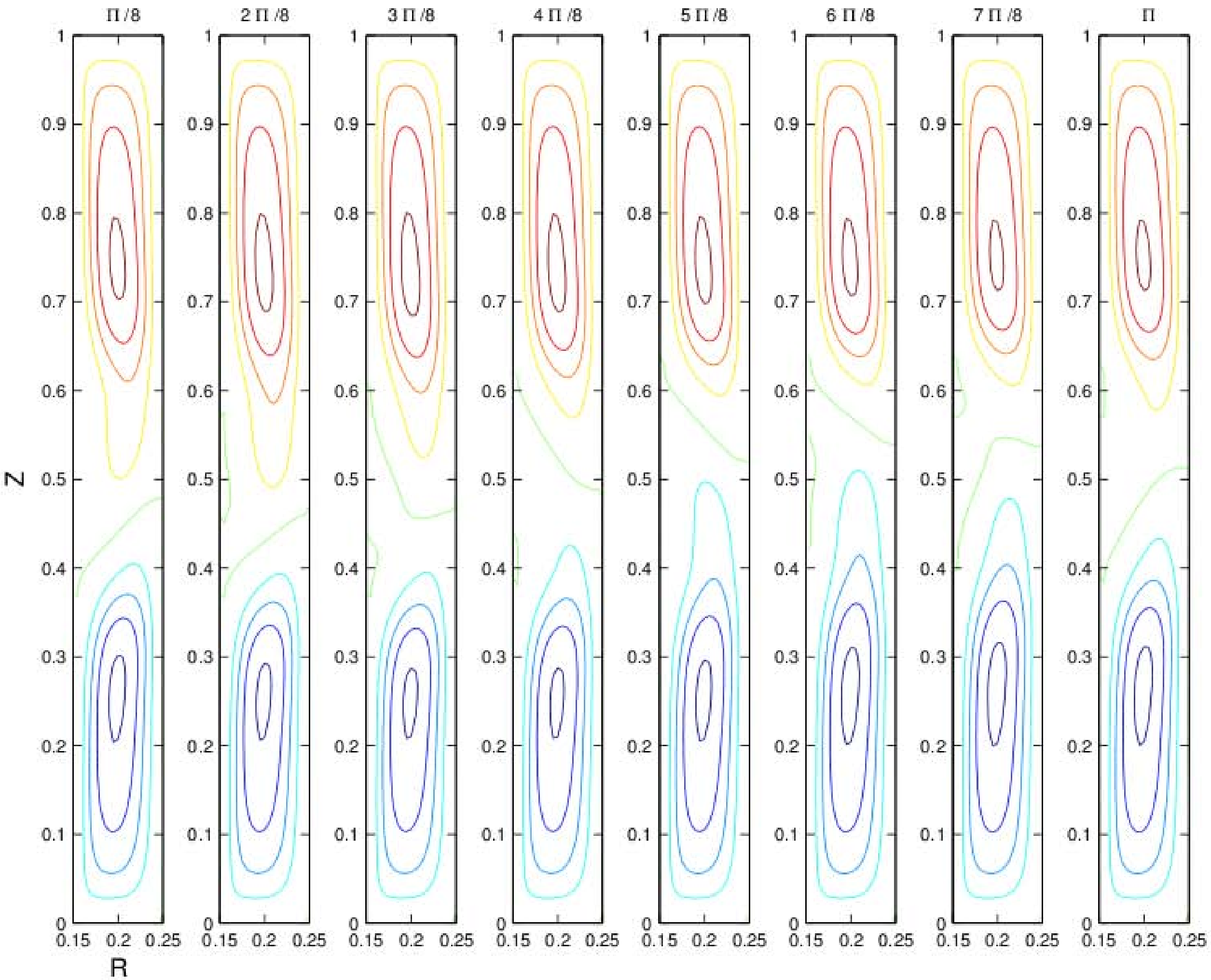


Fig. 2: Sequential contour plot of the stream function illustrating oscillation of the natural convection cells through one period (Π). $Ra = 2.5e7$, $A = 10$ and $\eta = 0.6$.

Simulation Results

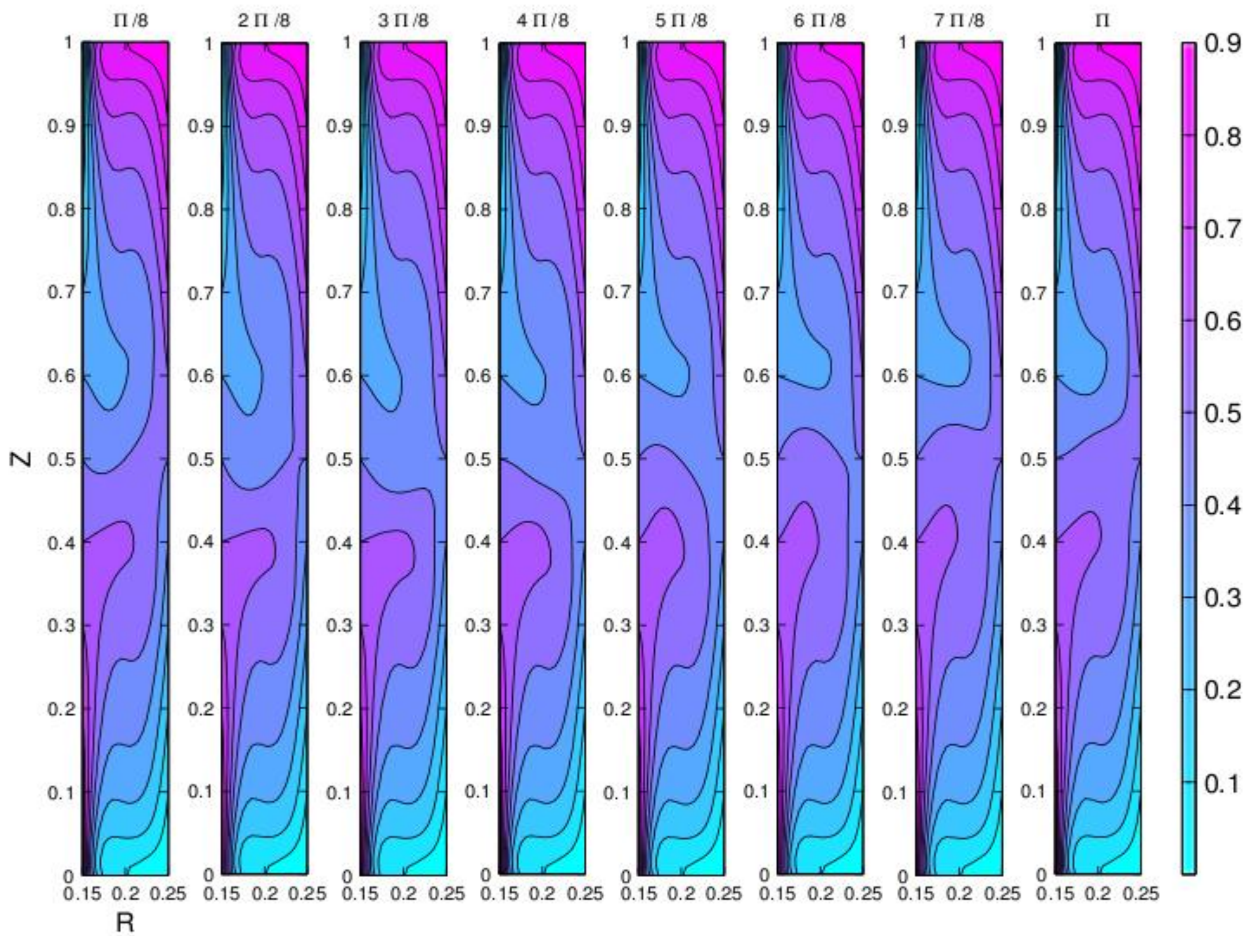


Fig. 3: Sequential contour plot of the temperature (Θ) illustrating oscillation of the natural convection cells through one period (Π). $Ra = 2.5e7$, $A = 10$ and $\eta = 0.6$.

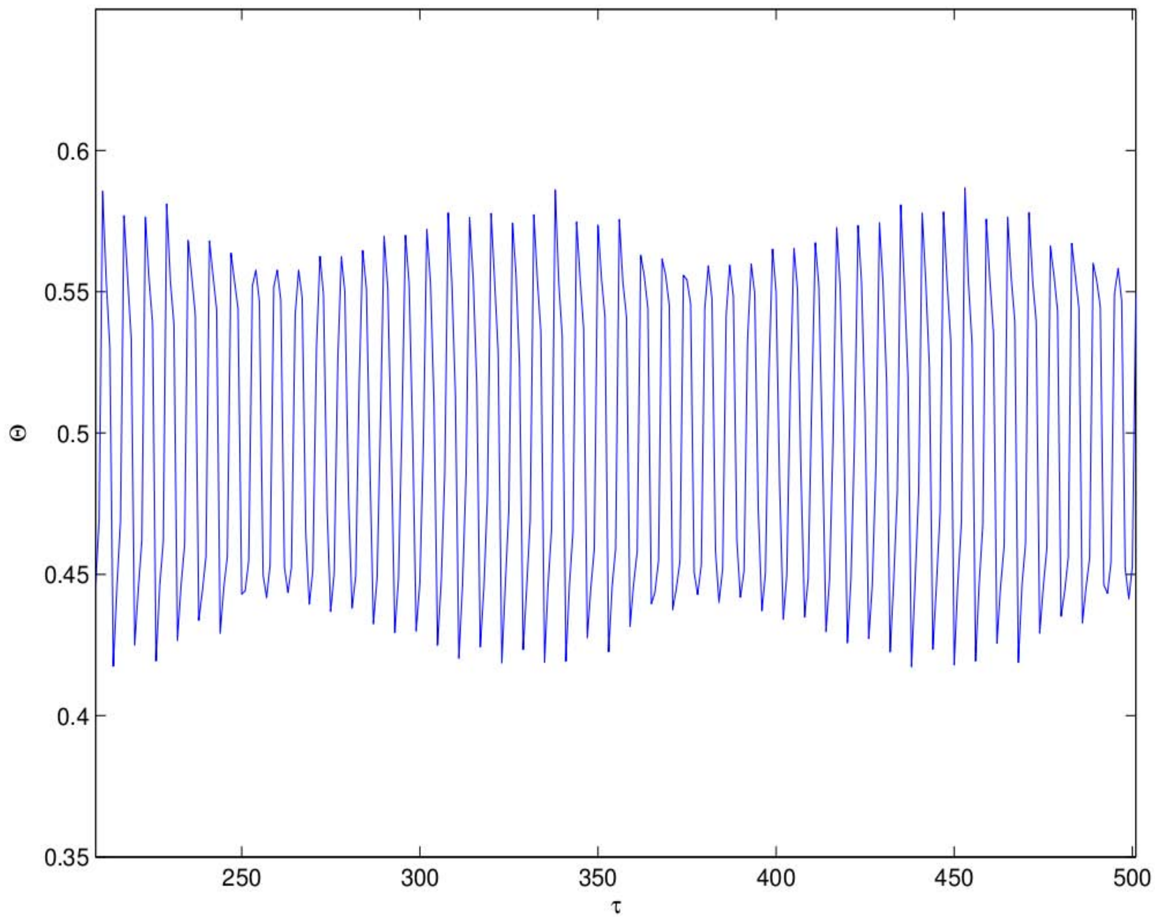
Dimensioned and Dimensionless Results

Description	Rayleigh	Density	k, cp, μ	Period	Amplitude
Option 1 [16]	2.5e7	Boussinesq	Constant	16.15	0.1285
Option 1	2.5e7	Boussinesq	Constant	16.126	0.1278
Dimensioned	2.5e7	Boussinesq	Constant	16.116	0.1279
Dimensioned	2.5e7	Ideal Gas	Constant	16.161	0.1277
Dimensioned	2.5e7	Ideal Gas	T dependent	16.161	0.1253
Option 1	4e7	Boussinesq	Constant	11.0703	0.1484
Dimensioned	4e7	Boussinesq	Constant	11.0091	0.1485
Dimensioned	4e7	Ideal Gas	Constant	11.0091	0.1485
Dimensioned	4e7	Ideal Gas	T dependent	11.0091	0.1481
Option 1	10e7	Boussinesq	Constant	7.1860	0.1361
Dimensioned	10e7	Boussinesq	Constant	7.0917	0.1362

Conclusions

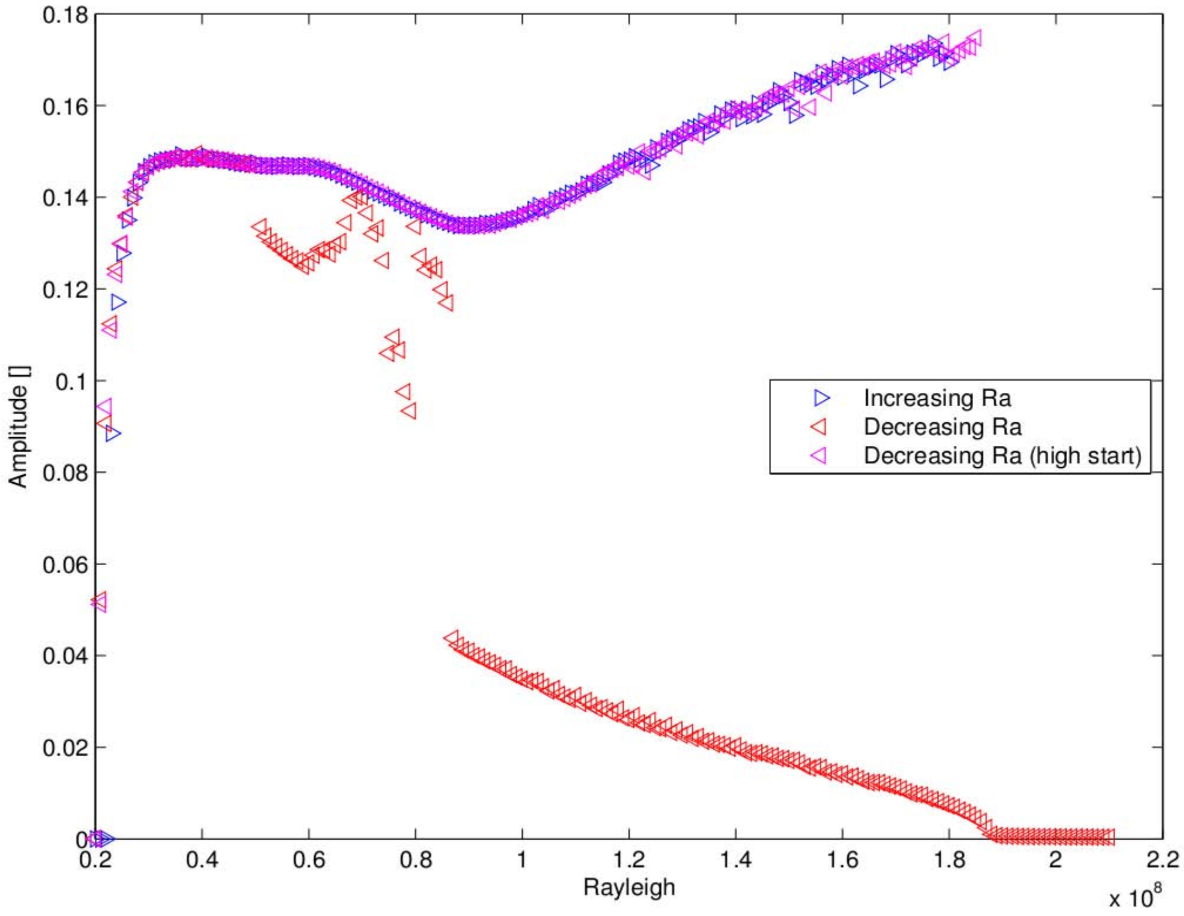
- Dimensioned and dimensionless solutions in COMSOL show good agreement for Ra=2.5e7 using the Boussinesq approximation.
- As the Rayleigh number is increased (Ra=10e7) the ideal gas solutions follow a separate solution path. This is a function of the chaotic behaviour of the system. This phenomena has also been observed at higher Rayleigh numbers when the Boussinesq approximation is used [16].

Chaotic behaviour of the System



At high Rayleigh numbers the system becomes chaotic as new harmonics appear. Ra=18e7 is shown.

Chaotic behaviour of the System



The system shows hysteresis. Depending on the starting point for the simulations different solution paths are found.

Future Work

- Continue exploration of the chaotic nature of this system.

Acknowledgements

Thanks to NSF for the principle funding through Grant 0626533.

Undergraduate student support from Jaeger Dill, Sarah Edwards, and Kimberley Hartman.